Personalized Pricing and Consumer Welfare [1]

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Abstract

We study the welfare implications of personalized pricing implemented with machine learning. We use data from a randomized controlled pricing field experiment to construct personalized prices and validate these in the field. We find that unexercised market power increases profit by 55%. Personalization improves expected profits by an additional 19%, and by 86%, relative to the non-optimized price. While total consumer surplus declines under personalized pricing, over 60% of consumers benefit from personalization. Under some inequity-averse welfare functions, consumer welfare may even increase. Simulations reveal a non-monotonic relationship between the granularity of data and consumer surplus under personalization.
Keywords: price discrimination, welfare, field experiment, personalized pricing, targeted marketing, machine learning, Lasso , Deep Learning, weighted likelihood bootstrap

1 Introduction

The vast quantities of personal data available to firms today have enormous economic potential. These data represent valuable business assets when firms use them to target decisions, like advertising and pricing, differentially across individuals. Recent events, such as the controversy over Cambridge Analytica's alleged misuse of user data on Facebook (Granville, 2018), the adoption of General Data Protection Regulation (hereafter GDPR) in the EU and the passage of the California Consumer Privacy Act (CCPA) of 2018, have created a surge in public interest and debate over acceptable commercial uses of consumer data. The data policies that have emerged, or are currently under debate, as a consequence of these events have restricted commercial uses of consumer data ostensibly to protect consumers and their privacy. However, the overall welfare implications of such privacy and data policies are not completely transparent and could have the unintended consequence of harming consumer surplus.

In this paper, we study the welfare implications of one particular controversial form of data-based decision-making: personalized pricing. Personalized pricing represents an extreme form of third-degree price discrimination that implements consumer-specific[1] prices using a large number of observable consumer features.[2] Prices are set differentially across each combination of observed consumer features to capture surplus. The application of modern machine learning tools enables firms to applysuch segmented pricing at scale.

The current extent of personalized pricing used in practice is unknown and "examples remain fairly limited" (CEA, 2015, p. 3).[3] In practice, third-degree price discrimination is still less common than second-degree price discrimination policies involving non-linear pricing schedules or menus of differentiated substitute productsMussa and Rosen (e.g., 1978). Nevertheless, growing public policy concern over the prospect of differential pricing on scale prompted a 2015 report by the Counsel of Economic Advisors (CEA) devoted entirely to differential pricing with "big data" (CEA, 2015). Recognizing how "...big data and electronic commerce have reduced the costs of targeting and first-degree price discrimination" (CEA, 2015, page 12), the report mostly drew dire conclusions about the potential harm to consumers:

> "[Differential pricing] transfers value from consumers to shareholders, which generally leads to an increase in inequality and can therefore be inefficient from a utilitarian standpoint" (CEA, 2015, page 6).

[1] Even though our B2B case study involves enterprise customers, we use the term "consumer" herein to refer to the buyers to conform with the terminology typically used in economics literature on demand-side welfare.

[2] In practice, third-degree price discrimination has typically been based on very coarse segmentation structures that vary prices across broad groups of consumers such as senior citizens' and children's discounts at the movies, and geographic or "zone" retail pricing by chain-stores across different neighborhoods within a metropolitan area. Only with the recent rise of the commercial internet and digitization has the potential for more granular, personalized segmentation structures become practical and scalable for marketing purposes (Shapiro and Varian, 1999; Smith, Bailey, and Brynjolfsson, 2000).

[3] Even large, digitally-enabled firms like Amazon have committed to an explicit, non-discriminatory pricing policy (Wolverton, 2010).

Similar concerns about the harmful effects of differential pricing have been echoed in the recent mainstream business media (e.g., Useem, 2017; Mohammed, October 20, 2017), leading experts to question the fairness and even legality of these practices (e.g., Krugman, October 4, 2000; Ramasastry, June 20, 2005; Turow, Feldman, and Meltzer, 2005). While the CEA report does not specifically recommend new legislation to regulate personalized pricing, privacy legislation, such as the GDPR, will require firms to disclose their usage of consumer data "in a concise, transparent, intelligible and easily accessible form, using clear and plain language.[4]" The GDPR may also require consumers to give consent before receiving personalized prices, which could limit the granularity of price discrimination and the types of variables firms are allowed to use when they set their prices. A similar set of clauses are also found in the recent California Consumer Privacy Act.

A potential concern is that over-regulation of data-based price discrimination could in fact have the unintended consequence of reducing social welfare and, more specifically, harming consumers in some situations. While it is well understood that, in a monopoly setting, price discrimination will typically benefit the firm, there is no general result as far as consumer welfare is concerned. The research in this area has derived, in a variety of settings, sufficient conditions on the shape of demand to determine whether third-degree price discrimination would increase social welfare (e.g., Pigou, 1920; Varian, 1989; Cowan and Vickers, 2010), and consumer welfare specifically (e.g., Cowan, 2012). In a more recent theoretical analysis Bergemann, Brooks, and Morris (2015) show that the consumer welfare implications of third-degree price discrimination depend on the attainable set of consumer segmentation structures using a firm's database. Unlike perfect price discrimination which transfers all the consumer surplus to the firm, personalized pricing often includes an element of classification error and, theoretically, could increase consumer surplus relative to optimal uniform pricing. Determining the extent to which "the combination of sophisticated analytics and massive amounts of data will lead to an increase in aggregate welfare" versus "mere changes in the allocation of wealth" has been identified as a fruitful direction for future research in the economics of privacy (Acquisti, Taylor, and Wagman, 2016, page 481).

To analyze the welfare implications of personalized pricing, we conduct an empirical case study in cooperation with a large digital firm that was in the early stages of re-examining its pricing policy. The heart of our analysis consists of a sequence of novel, randomized business-to-business price experiments for new consumers. In the first experiment, we randomize the quoted monthly price of service to new consumers and use the data to train a demand model with heterogeneous price treatment effects. We assume that the heterogeneity in consumers' price sensitivities can be characterized by a sparse subset of an observed, high-dimensional vector of observable consumer features. The demand estimates allow us to design an optimized uniform pricing structure and an optimized personalized pricing structure. We use a Bayesian Decision-Theoretic formulation of the

[4] Article 12, EU GDPR, "Transparent information, communication and modalities for the exercise of the rights of the data subject".

firm's pricing decision problem (Wald, 1950; Savage, 1954), defining the posterior expected profits as the reward function to account for statistical uncertainty. In a second experiment with a new sample of consumers, we then test our model pricing recommendations and inference procedure out of sample, a novel feature of our analysis (see also Misra and Nair 2011; Ostrovsky and Schwarz 2016[5]).

To the best of our knowledge, this study is the first to document both the feasibility and implications of scalable personalized pricing. In this regard, we add to a small and growing literature using firm-sanctioned field experiments to obtain plausible estimates of the treatment effect of marketing variables on demand (e.g., Levitt and List, 2009; Einav and Levin, 2010) [6]. The fact that our corporate partner, Ziprecruiter, has authorized us to disclose its identity and the details of the underlying experiment also supports the growing importance of transparency and disclosure when using firm-sponsored experiments for scientific research (Einav and Levin, 2014).

While not the main focus of the paper, the field experiment reveals a striking degree of unexercised market power. The data-based, optimal uniform price is 230% higher than the firm's status quo pricing, an opportunity to increase profits by 55%. These large price and profit improvements are robust to the optimization of longer-term discounted profits that also account for future consumer retention rates. The second experiment confirms the profit increases from data-based pricing out of sample. In fact, shortly after the first experiment, Ziprecruiter permanently increased its price to $249, at least until as recently as November of 2020.[7]

Our demand estimates also reveal a considerable degree of heterogeneity in willingness-to-pay. We predict that decision-theoretic personalized pricing would increase the firm's posterior expected profits by 86% relative to its status quo price of $99, and by 19% relative to the decision-theoretic optimal uniform price of $327. These predicted profit improvements are robust to a longer-term time horizon of several months. We validate the predicted profit gains out of sample using our second experiment. Although the gains in profits are not surprising theoretically, the magnitudes are considerably higher than those predicted in past work using observable consumer variables (Rossi, McCulloch, and Allenby, 1996; Shiller and Waldfogel, 2011; Shiller, 2015).

On the demand side, the evaluation of consumer welfare in a setting without a representative-consumer formulation requires the specification of a social welfare function. Under a total consumer surplus standard, we predict that consumer welfare would fall under decision-theoretic personalized pricing relative to optimal uniform pricing. In this regard, our findings confirm some of the concerns about consumer harm in the public policy debate. But, for our case study, personalization is still far removed from the purely theoretical case of perfect price discrimination which transfers all the consumer surplus to the firm. Simulations based on the estimates from the first experiment

[5]Misra and Nair (2011) test the performance of a more efficient incentives-based compensation scheme for sales agents in a large firm, and Ostrovsky and Schwarz 2016 test the performance of optimally-derived reserve prices for Yahoo!'s sponsored search auctions

[6]See also Cohen, Hahn, Hall, Levitt, and Metcalfe (2016) for a quasi price experiment based on Uber surge.

[7]More recently, the firm has implemented a menu of prices that includes $249 as the price of the base product.

predict that the majority of consumers benefit from personalization relative to the optimal uniform price, indicating redistributive benefits albeit at the expense of the highest willingness-to-pay consumers. In our second validation field experiment, nearly 70% of the consumers assigned to the personalized pricing cell are targeted a personalized price that is below the optimal uniform price. Under alternative inequality-averse consumer welfare functions (Lewbel and Pendakur, 2017; Atkinson, 1970; Jorgenson, 1990), we find that these redistributive benefits could outweigh the losses in total consumer surplus depending on the degree of the social planner's inequality aversion. Although our experiments are not designed to identify the causal effect of specific individual consumer features on demand, in an exploratory exercise, we find that the "firm size" and "benefits offered to employees" features are the most highly correlated with incidence of receiving a personalized price below the uniform rate. Therefore, personalization appears to benefit smaller and more disadvantaged firms; albeit at the cost of an overall decrease in total consumer surplus. Our results do not appear to be an artifact of the the use of a standard LASSO regularization algorithm. Qualitatively, our findings are robust to a recently-developed, alternative deep learning approach developed by Farrell, Liang, and Misra (2021b) and Farrell, Liang, and Misra (2021a).

The main focus of our analysis is on the use of our model estimates to explore the role of the granularity of consumer information on surplus. We examine several alternative personalization schemes that restrict the types of consumer features on which the firm is allowed to condition to construct segments and set differential prices. Consistent with Bergemann, Brooks, and Morris (2015), we find a non-monotonic relationship between consumer surplus and the quantity of consumer data available to the firm for personalization. While all of our personalization scenarios generate less consumer surplus than uniform optimal pricing, we find several cases where restricting the firm's information set leads to even less consumer surplus in spite of the coarsening of the segments. This non-monotonicity is also robust to the use of the deep learning algorithm. This empirical finding that consumer surplus is non-monotonic in the degree of consumer information suggests that any regulation of consumer data might need to consider carefully the welfare implications caused by "downstream" decisions based on such data.

Our findings contribute to the empirical literature on third-degree price discrimination (see the survey by Verboven 2008). The price experiment avoids the typical price endogeneity concerns associated with demand estimation based on observational data and offers a clean study of the impact of third-degree price discrimination on the firm's outcomes. In the domain of digital marketing, Bauner (2015) and Einav, Farronato, Levin, and Sundaresan (2017) argue that the co-existence of auctions and posted price formats on eBay may be a sign of price discrimination across consumer segments. Einav, Farronato, Levin, and Sundaresan (2017) conclude that "richer econometric models of e-commerce that incorporate different forms of heterogeneity ... and might help rationalize different types of price discrimination would be a worthwhile goal for future research." In a large-scale randomized price experiment for an online gaming company that uses almost uniform pricing, Levitt, List, Neckermann, and Nelson (2016) find almost no effect on revenues from various alternative second-degree "non-linear" price discrimination policies. However, they document substantial heterogeneity across consumers which suggests potential gains from the type of third-degree "personalized pricing" studied herein. Subsequent to the writing of this paper, Kehoe, Larsen, and Pastorino (2020) also analyze the potential consumer welfare-increasing effects of personalized pricing in a dynamic durable-goods duopoly market.

Our work also contributes to the broader empirical literature on the targeting of marketing actions across consumers (e.g., Ansari and Mela, 2003; Simester, Sun, and Tsitsiklis, 2006; Dong, Manchanda, and Chintagunta, 2009; Kumar, Sriram, Luo, and Chintagunta, 2011). A small subset of this literature has analyzed personalized pricing with different prices charged to each consumer (e.g., Rossi, McCulloch, and Allenby, 1996; Chintagunta, Dubé, and Goh, 2005; Zhang, Netzer, and Ansari, 2014; Waldfogel, 2015; Shiller, 2015). Our work is closest to Shiller (2015) who also uses machine learning to estimate heterogeneous demand. Most of this research uses a retrospective analysis of detailed consumer purchase histories to determine personalized prices. These studies report large predicted profit improvements for firms when they target on consumers' historic purchase behavior. However, the implications for targeted pricing are typically studied through model simulations based on demand estimates. In contrast, we run field experiments, not only to estimate demand, but also to provide an out-of-sample field validation of the our model predictions for the impact on consumers and the firm. The extant work's findings and methods also have limited applicability beyond markets for fast-moving consumer goods due to the limited availability of consumer purchase panels in most markets. In contrast, we devise a more broadly practical targeting scheme based on observable consumer features and cross-sectional data.

The extant literature suggests that basing personalized prices on observable consumer features, as opposed to purchase histories, generates modest gains for firms, casting doubts on the likelihood that firms would invest in implementing such pricing practices. For example, Rossi, McCulloch, and Allenby (1996) conclude that "...it appears that demographic information is only of limited value" for the personalization of prices of branded consumer goods. Similarly, Shiller and Waldfogel (2011) claim that "Despite the large revenue enhancing effects of individually customized uniform prices, forms of third degree price discrimination that might more feasibly be implemented produce only negligible revenue improvements." In the internet domain, Shiller (2015) finds "...demographics alone to tailor prices raises profits by 0.8% [at Netflix]." These findings may explain the lack of empirical examples of large-scale personalized pricing in practice. One exception is List (2004) who finds that sports-card dealers actively use minority status as a for proxy differences in consumer willingness-to-pay, though he does not explore the profit implications. In contrast, our findings suggest that personalized pricing based on observable consumer features could improve firm profits substantially, supporting the view that such practices could become more commonplace.

Our findings also relate to the concept of fairness in the social choice literature and add to the on-going public policy debate regarding the "fairness" aspects of differential pricing. In our discrete-choice demand setting, only a uniform pricing policy would satisfy the "no envy" criterion of the fair allocations studied in the social choice literature (Foley, 1967; Thomson, 2011). Absent wealth transfers, in our case study this "fair" outcome could lead to fewer served consumers and lower consumer surplus, highlighting the potential trade-offs between fairness and consumer welfare. Moreover, in our case study the typical strong consumer tends to be a larger company with 20 employees (relative to 10 employees for weak consumers), suggesting that our personalization scheme redistributes surplus from larger to smaller consumers. This type of reallocation could be rationalized as "fair" under a Pareto-weight scheme that assigns higher social value to smaller, disadvantaged firms. In this regard, our findings also contribute to the emerging literature on the economics of privacy (e.g., Acquisti, Taylor, and Wagman, 2016) by documenting potential benefits to consumers from personalization.

The remainder of the paper is organized as follows. In section 2, we set up the prototypical decision-theoretic formulation of monopoly price personalization based on demand estimation. In section 3, we derive our empirical approach for estimating the demand parameters and quantifying uncertainty. We summarize our empirical case study of targeted pricing at Ziprecruiter.com in section 4. We conclude in section 7.

2 A Model of Decision-Theoretic Monopoly Price Personalization

In this section, we outline the key elements of a data-based approach to monopoly price discrimination. We cast the firm's pricing decision as a Bayesian statistical decision theory problem (e.g., Wald 1950; Savage 1954; Berger 1985 and also see Hirano 2008 for a short overview along with Green and Frank 1966 and Bradlow, Lenk, Allenby, and Rossi 2004 for a discussion of Bayesian decision theory for marketing problems). The firm trades off the opportunity costs from sub-optimal pricing and the statistical uncertainty associated with sales and profits at different prices. We cast the firm's uncertainty as a lack of precise statistical information about an individual consumer's preferences and demand. Bayes theorem provides the most appropriate manner for the firm to use available data to update its beliefs about consumers and make informed pricing decisions. Failure to incorporate this uncertainty into pricing decisions could lead to bias, as we discuss below. We also discuss herein the potential short-comings of a simpler approach that "plugs in" point estimates of the uncertain quantities instead of using the full posterior distribution of beliefs. For an early application of Bayesian decision theory to pricing strategy see Green (1963). For a more formal econometric treatment of Bayesian decision-theoretic pricing that integrates consumer demand estimation, see Rossi, McCulloch, and Allenby (1996); Dubé, Fang, Fong, and Luo (2017)[8].

We start by describing the demand setup and defining the sources of statistical uncertainty

[8]See Hitsch (2006) for an application of Bayesian decision-theoretic sequential experimentation.

regarding consumers and their demand. The demand model represents the firm's prior beliefs about the consumer. On the supply side, we then define the firm's information set about the consumer. By combining the firm's prior beliefs (the demand model) and available information (the consumer data), we then define several decision-theoretic (or "data-based") optimal pricing problems for the firm.

2.1 Demand

Below we present a relatively agnostic, multi-product derivation of demand to illustrate the generalizability of our approach across a wide class of empirical demand settings. Consider a population of $i = 1, ...H$ consumers. Each consumer i chooses a consumption bundle $q = (q_1, ..., q_J) \in \mathbb{R}_+^J$ to maximize her utility as follows:

$$\bar{q}\left(p_i; \Psi_i, \epsilon_i\right) = \arg\max_q \left\{U\left(q; \Psi_i, \epsilon_i\right) \ : \ p_i' q \leqslant I\right\} \tag{1}$$

where $U\left(q; \Psi_i, \epsilon_i\right)$ is continuously differentiable, strictly quasi-concave and increasing in q, I is a budget, $p_i = (p_{i1}, ..., p_{iJ}) \in \mathbb{R}_+^J$ is the vector of prices charged to consumer i, Ψ_i represents consumer $i's$ potentially observable "type" (or preferences) and $\epsilon_i \sim i.i.d.\ F_\epsilon\left(\epsilon\right)$ is an i.i.d. random vector of unobserved, random disturbances that are independent of Ψ_i. In our analysis below, we distinguish between the aspects of demand about which a firm can learn, Ψ_i, and about which it cannot learn, ϵ_i.

2.2 Firm Beliefs and Pricing

We now define the personalized pricing problem and its relationship to the price discrimination literature. To capture the marketplace realities of data-based marketing, we model the firm's design of personalized pricing as a statistical decision problem.

Suppose the firm knows the form of demand, 1, and has prior beliefs about Ψ_i described by the density $f_\Psi\left(\Psi_i\right)$. Let $\boldsymbol{D}$ denote the consumer database collected by the firm. We assume the firm uses Bayes Rule to construct the data-based posterior belief about the consumer's type:

$$f_\Psi\left(\Psi_i|\boldsymbol{D}\right) = \frac{\ell\left(\boldsymbol{D}|\Psi_i\right) f_\Psi\left(\Psi_i\right)}{\int \ell\left(\boldsymbol{D}|\Psi_i\right) f_\Psi\left(\Psi_i\right) d\Psi_i} \tag{2}$$

where $\ell\left(\boldsymbol{D}|\Psi_i\right)$ is the log-likelihood induced by the demand model, 1 and the uncertainty in the random disturbances, ϵ_i. Let $F_\Psi\left(\Psi_i|\boldsymbol{D}\right)$ denote the corresponding CDF of the posterior beliefs. Note that we assume the firm does not update its beliefs $F_\epsilon\left(\epsilon\right)$ about the random disturbances, ϵ_i.

Given the posterior $F_\Psi\left(\Psi_i|\boldsymbol{D}\right)$, the firm makes decision-theoretic, data-based pricing decisions. We assume the firm is risk neutral and faces unit costs $c = (c_1, ..., c_J)$ for each of its products. For each consumer i, the firm anticipates the following posterior expected profits from charging prices p_i :

$$\pi\left(p_i|\mathbf{D}\right) = \left(p_i - c\right)' \int\int \bar{q}\left(p;\Psi_i,\epsilon\right) dF_\epsilon\left(\epsilon\right) dF_\Psi\left(\Psi_i|\mathbf{D}\right). \tag{3}$$

The firm's optimal *personalized prices* for consumer i, p_i^*, must therefore satisfy the following first-order necessary conditions:

$$p_i^* = c - \left[\int\int \nabla_p \bar{q}\left(p_i^*;\Psi_i,\epsilon\right) dF_\epsilon\left(\epsilon\right) dF_\Psi\left(\Psi_i|\mathbf{D}\right)\right]^{-1} \int\int \bar{q}\left(p_i^*;\Psi_i,\epsilon\right) dF_\epsilon\left(\epsilon\right) dF_\Psi\left(\Psi_i|\mathbf{D}\right) \tag{4}$$

where $\nabla_p \bar{q}\left(p_i^*;\Psi_i,\epsilon\right)$ is the matrix of derivatives of consumer $i's$ demand with respect to prices. If the firm instead implements a *uniform* pricing strategy across all its H consumers, the posterior expected profit-maximizing uniform prices, p^*, must satisfy the following first-order necessary conditions:

$$p^* = c - \left[\sum_i^H \int\int \nabla_p \bar{q}\left(p^*;\Psi_i,\epsilon\right) dF_\epsilon\left(\epsilon\right) dF_\Psi\left(\Psi_i|\mathbf{D}\right)\right]^{-1} \sum_i^H \int\int \bar{q}\left(p^*;\Psi_i,\epsilon\right) dF_\epsilon\left(\epsilon\right) dF_\Psi\left(\Psi_i|\mathbf{D}\right). \tag{5}$$

The recent public policy debate regarding consumer data and targeted pricing has frequently associated personalized pricing with traditional first-degree price discrimination. While *first-degree* or *perfect price discrimination* has typically been viewed as a polar, theoretical case (e.g. Pigou, 1920; Varian, 1980; Stole, 2007; Bergemann, Brooks, and Morris, 2015), theorists have long recognized the possibility that with a very granular segmentation scheme, third-degree price discrimination could approximate first-degree price discrimination[9]:

> "... it is evident that discrimination of the third degree approximates towards discrimination of the first degree as the number of markets into which demands can be divided approximate toward the number of units for which any demand exists." (Pigou, 1920, Part II, chapter XVI, section 14)

In fact, the personalized pricing in (4) technically constitutes a form of *third-degree price discrimination* (e.g. Tirole, 1988; Pigou, 1920). In our model, the firm can never learn ϵ_i even with repeated observations on the same consumer (i.e. panel data). Therefore it will never be possible for the firm to extract all of the consumer surplus even when all the uncertainty in Ψ_i is resolved. In practice, the prices are not fully personalized since consumers with the same posterior expected Ψ_i would always be charged the same price even if they differ along unobserved dimensions.

[9] Statistical uncertainty typically limits the segmentation to an imperfect form of targetability. The approximation is also typically closer under unit demand since personalization typically cannot target a different price to each infra-marginal unit purchased by a consumer.

2.3 Welfare

2.3.1 Welfare Implications of personalization

At the heart of the public policy debate is a wide-spread belief that machine learning and databased marketing will harm consumers *per se*. Monopoly personalized pricing will always weakly increase the firm's profits since, by revealed preference, the firm can always choose to charge every consumer the same uniform price in 5: $p_i^* = p^*$, $\forall i$.[10] The predicted impact of personalized prices on consumer surplus is less straightforward. Under perfect price discrimination, the monopolist extracts all the consumer surplus. As consumer data converges to the point where a firm can perfectly predict a consumer's willingness to pay for each marginal unit, we would expect additional information to reduce consumer surplus per se. But, perfect price discrimination is at best a theoretical polar case. Even in fast-moving consumer goods industries where the firm can track the same consumer's shopping choices repeatedly over time, potentially at different prices, researchers still observe a substantial amount of random (unpredictable) switches in consumer choices (e.g., Rossi, McCulloch, and Allenby, 1996). Therefore, for the foreseeable future, personalized pricing will at best achieve an extremely granular form of third, as opposed to first, degree price discrimination.

The extant literature on monopoly third-degree price discrimination has relied on local conditions regarding the curvature of demand and other regularity conditions to determine the impact on social surplus (e.g., Varian, 1989) and consumer surplus specifically (e.g., Cowan, 2012). More recently, Bergemann, Brooks, and Morris (2015) show that, theoretically, third-degree price discrimination "can achieve every combination of consumer surplus and producer surplus such that: (i) consumer surplus is nonnegative, (ii) producer surplus is at least as high as profits under the uniform monopoly price, and (iii) total surplus does not exceed the surplus generated by efficient trade." Therefore, the impact of the personalized prices characterized by 4 on consumer surplus is ultimately an empirical question about the segments constructed with the database $\mathbf{D}$.

To illustrate this point, consider a market with six consumers $\{i\}_{i=1}^{6}$ with valuations $\Psi_i = \$i$. Assume that costs are negligible (close to zero) and are relevant only as tie-breakers between profit-equivalent choices. In Table 1, we report the results under several information scenarios. Under perfect price discrimination, the firm charges each consumer her valuation, generating \$21 in profits and \$0 consumer surplus. Under a profit maximizing uniform pricing policy, the firm charges $p_i = \$4\ \forall i$ which generates \$12 in profits and \$3 in consumer surplus.[11] Total surplus however is only \$15 and there is a deadweight loss of \$6.

Now, suppose the firm has a database, $\mathbf{D}$, that signals information about consumers' types, allowing it to distinguish between the following two segments: $\{1\}$ and $\{2, 3, 4, 5, 6\}$. Under third-

[10] We make the usual assumption of no-arbitrage between consumers.

[11] The firm does not charge a uniform price equal to \$3 because of our assumption of a small, but positive marginal cost to break the tie between \$3 and \$4.

degree price discrimination, the firm can increase it profits to \$13 by charging the segment prices: $p_{\{1\}} = \$1$ and $p_{\{2,3,4,5,6\}} = \$4$. In this case, consumer surplus remains fixed at \$3. Total surplus however has increased by \$16 and the deadweight loss is now only \$5.

Now consider the more granular database, $\tilde{\mathbf{D}}$, that allows the firm to classify the consumers into the following three segments: $\{1\}, \{2,3\}$ and $\{4,5,6\}$. For instance, suppose that a change in public policy that previously protected the identity of consumers 2 and 3 (e.g., race or gender) is relaxed, allowing the firm to target this segment with differential prices. Under third-degree price discrimination, the firm can now increase it profits to \$17 by charging the segment prices: $p_{\{1\}} = \$1$,$p_{\{2,3\}} = \$2$ and $p_{\{4,5,6\}} = \$4$. As we increase the granularity of the database and allows for more personalized pricing, consumer surplus increases to \$4. Moreover, total surplus is now \$21, which is equal to total surplus under perfect price discrimination except that some of the value accrues to consumers. Interestingly, there is no deadweight loss in this case.

These findings are robust to the inclusion of classification error (i.e., an un-targetable type I extreme value random utility shock): $\Psi_i = i + \epsilon_i$. In this case, both segmentation schemes increase both firm profit and consumer surplus relative to the uniform pricing scenario. However, the granular database increases consumer surplus by less than the coarse database, indicating a non-monotonicity in the relationship between consumer surplus and the degree of granularity of the segmentation scheme.

This example merely illustrates that increasing the granularity of the consumer data available to a firm can increase consumer surplus and even reduce deadweight loss. Obviously, there are other databases that could lead to segmentation schemes that would have different welfare implications. But the example indicates that the consumer welfare implication of personalized pricing is ultimately an empirical question that depends on the databases available to firms for marketing decision-making. In the next section, we discuss how a firm can use large consumer databases and machine-learning to construct scalable segmentation schemes.

2.3.2 Welfare Aggregation

The discussion above assumes that society values only the total consumer surplus, with no weight assigned to the allocation. This perspective is reflected in the commonly used linear aggregation of total consumer surplus as a welfare measure

$$S(\mathbf{p}) = \frac{1}{N} \sum \left(V(\mathbf{p}, x_i) \right), \tag{6}$$

where $\mathbf{p} = \{\widetilde{p}_i\}_{i=1}^N$ is the vector of prices charged to consumers, $V_i(\mathbf{p})$ denotes consumer i's realized surplus in dollars.[12] This measure of surplus fails to account for any distributional effects besides the average. In his classic IO textbook, Tirole describes the limitations of this approach as follows:

[12] In our empirical case study below, we follow the convention in the empirical literature and approximate $V_i(\mathbf{p})$ using the Hicksian compensating variation.

"...the government has efficiency concerns but no redistribution concerns. Of course, one of the main policy issues in regard to price discrimination is its effect on income distribution." (Tirole, 1988, p. 139)

To account for distributional effects, we examine alternative aggregation metrics and following Lewbel and Pendakur (2017), consider a range of welfare functions derived from Atkinson (1970)'s mean of order r class:

$$S_r(\mathbf{p}) = \begin{cases} \left[\frac{1}{N}\sum (V_i(\mathbf{p}))^r\right]^{1/r}, & \text{for } r \neq 0 \\ \\ \exp\left(\frac{1}{N}\sum \ln V_i(\mathbf{p})\right), & \text{for } r = 0. \end{cases} \tag{7}$$

In equation (7), r determines society's preferences over allocations of surplus. The special case $r = 1$ (arithmetic mean) nests the commonly-used linear aggregation scheme in (6) above and reflects an inequality-neutral societal preference. As in Lewbel and Pendakur (2017), we focus on $r = -1, 0, 1$. The cases where $r \in \{-1, 0\}$ (harmonic and geometric mean) correspond to inequality-averse welfare functions that may select a personalized pricing policy that reduces total consumer surplus but, at the same time, disproportionately reduces inequality. This form of the welfare function is closely related to those proposed by Jorgenson (1990) and Jorgenson and Slesnick (2014) who also consider various generalized mean definitions to aggregate consumer surplus and evaluate the allocation.

3 Empirical Approach

The execution of the firm's data-based pricing strategies in equations 4 and 5 depends on the ability to construct an estimate of the posterior distribution $F(\Psi_i|\mathbf{D})$. The extant literature on price discrimination has developed non-linear panel data methods to estimate $F(\Psi_i|\boldsymbol{D})$ using repeated purchase observations for each consumer panelist (e.g. Rossi, McCulloch, and Allenby 1996; Chintagunta, Dubé, and Goh 2005). In practice, many firms do not have access to panel databases. In many business-to-business and e-commerce settings, for instance, firms are more likely to have access to data for a broad cross-section of consumers, but not with repeated observations.[13] We consider a scenario with cross-sectional consumer information that includes a detailed set of observable consumer features. Our approach consists of using these features to approximate Ψ_i.

[13]Ideal panel data would allow the firm estimate types using fixed effects estimators but there would remain the issue of pricing to new consumers which is our focus here.

3.1 Approximating Individual Types

Suppose we observe data

$$\mathbf{D}=\{(q_i,x_i,p_i)\}_{i=1}^{N}$$

for a sample of N consumers, where $q_i \in \mathbb{R}_+^J$ is a vector of purchase quantities, $p_i \in \mathbb{R}_+^J$ are the prices and $x_i \in \mathcal{X} \subseteq \mathbb{R}^K$ is a vector of consumer characteristics. We assume that x_i is high-dimensional and fully characterizes the preferences, Ψ_i. We consider the projection of the individual tastes, Ψ_i, onto x_i:

$$\Psi_i = \Psi\left(x_i;\Theta_0\right)$$

where Θ_0 is a vector of parameters. Note that for our pricing problem in section 2.2 above, we are not interested in the interpretation of the arguments of the function$\Psi\left(x_i;\Theta\right)$. So we could be agnostic with our specification. For instance, we could represent the function $\Psi\left(x_i;\Theta\right)$ as a series expansion:

$$\Psi\left(x_i;\Theta_0\right)=\sum_{s=1}^{\infty}\theta_{0s}\psi_s\left(x_i\right)$$

where $\{\psi_n\left(x_i\right)\}_{n\geq 0}$ is a set of orthonormal basis functions and $\Theta_{n0}=(\theta_1,...,\theta_n)$ are the parameters for an expansion of degree n. We are implicitly assuming that some sparse subset of the vector x_i is informative about Ψ_i and that we posses some methods to identify this sparse subset.

We focus on applications where K is large(potentially, $K \gg N$) and Θ_{n0} is relatively sparse. Even though our approach consists of a form of third-degree price discrimination, in practice, it can capture very rich patterns of heterogeneity. We assume the firm has a very high-dimensional direct signal about demand, x. For instance, if the dimension of x_i is $K=30$, our approach would allow for as many as $2^K=1,073,741,824$ distinct consumer types and, potentially, personalized prices.

3.2 Approximating $F\left(\Psi_i|\mathbf{D}\right)$: The Weighted Likelihood Bootstrapped Lasso

With $K \gg N$, maximum likelihood is infeasible unless one has a theory to guide the choice of coefficients to include or exclude. Even in cases where K is large and $K<N$, maximum likelihood could be problematic and lead to over-fitting. The literature on regularized regression provides numerous algorithms for parameter selection with a high-dimensional parameter vector, Θ (e.g. Hastie, Tibshirani, and Friedman, 2009). Most of this literature is geared towards prediction. Our application requires us to quantify the uncertainty around our estimated coefficient vector, $\hat{\Theta}$, and around various economic outcomes such as price elasticities, firm profits and consumer value, to implement decision-theoretic optimized pricing structures. In addition, the approach must be fast enough for real-time demand forecasting and price recommendations.

Our framework conducts rational Bayesian updating with the goal of obtaining the posterior distribution of interest using a *loss* function, as opposed to a likelihood function. Bissiri, Holmes, and Walker (2016) show that for a prior, $h(\Theta)$, data, $\mathbf{D}$, and some loss function $l(\Theta, \mathbf{D})$, the object $f(\Theta|\mathbf{D})$ defined by

$$f(\Theta|\mathbf{D}) \propto \exp(-l(\Theta, \mathbf{D}))\, h(\Theta) \tag{8}$$

represents a coherent update of beliefs under loss function $l(\Theta, \mathbf{D})$. As such, it represents posterior beliefs about the parameter vector Θ given the data as encoded by the loss function $l(\Theta, \mathbf{D})$. In our setting, we specify the loss function as a L_1 penalized (Lasso) negative log-likelihood:

$$l(\Theta, \mathbf{D}) = -\left[\sum_{i=1}^{N} \ell(\mathbf{D}_i|\Theta) - \lambda \sum_{j=1}^{J} |\Theta_j|\right] \tag{9}$$

where $\sum_{i=1}^{N} \ell(\mathbf{D}_i|\Theta)$ is the sample log-likelihood induced by the demand model in section 1, and λ is a penalization parameter.

We then approximate the posterior $F_\Psi(\Psi|\mathbf{D})$ using a variant of the Bayesian Bootstrap (e.g., Rubin, 1981; Newton and Raftery, 1994; Chamberlain and Imbens, 2003; Efron, 2012). In particular, we simulate draws from the the posterior distribution of the model parameters using a weighted likelihood bootstrap algorithm (WLB) as outlined in Newton and Raftery (1994).[14] The approach we follow is similar to the "loss likelihood bootstrap" outlined in Lyddon and Holmes (2019) who also derive the large-sample properties for these estimators. Broadly speaking, our procedure operates by assigning weights, drawn from a Dirichlet distribution, to each observation and implementing the Lasso estimator that conditions on these weights. Repeating this B times gives us an approximate sample from the full posterior distribution $F_\Psi(\Psi|\mathbf{D})$ which can be used to compute the posterior distribution and other derived quantities required for the decision-theoretic pricing problem.

Formally, our estimator consists of B replications of the following weighted-likelihood Lasso regression, where at step b:

$$\hat{\Theta}^b = \underset{\Theta \in \mathbb{R}^J}{\arg\max} \left\{ \sum_{i=1}^{N} V_i^b \ell(D_i|\Theta) - N\lambda \sum_{j=1}^{J} |\Theta_j| \right\}.$$

We show in Appendix B that weights $V_i \sim$ i.i.d. $\text{Exp}(1)$ are equivalent to Dirichlet weights. Our procedure does not provide draws from the exact posterior and consequently $\left\{\hat{\Theta}^b\right\}_{b=1}^{B}$ should be treated as an approximate sample from the posterior of interest. One interpretation of our approach is that it represents the draws from the posterior that minimizes the Kullback-Leibler divergence between the parametric class we adopt and the true data generating process. This framework is coherent from a Bayesian perspective in spite of the non-standard implementation.

[14] For a detailed description of our procedure see Appendix B.

We refer the reader to Bissiri, Holmes, and Walker (2016) and Lyddon and Holmes (2019) for a more thorough discussion.

Our proposed algorithm deals with two sources of uncertainty simultaneously. In particular, by repeatedly constructing weighted Lasso type estimators we are in effect integrating over the model space spanned by the set of covariates. As such, our draws can also be used to construct posterior probabilities associated with the set of covariates retained in the model. At the same time, the sampling procedure also accounts for usual parameter uncertainty. An additional advantage of using the loss-likelihood approach is that we do not have to make parametric assumptions about our priors over the model space, allowing for additional robustness of our results. Subsequent to our analysis, new research has emerged with formal results on the sampling properties of similar machine-learning estimators applied to settings with high-dimensional observed heterogeneity (Athey and Imbens, 2016b,a). In our analysis below, we compare our findings with the WLB to a more sophisticated, non-parametric *deep learning* algorithm Farrell, Liang, and Misra (2021b,a). See Appendix D for details of the deep learning algorithm. Due to the binary nature of most of our consumer feature variables, this deep learning algorithm produces qualitatively similar results to WLB.

The extant literature has often followed a two-step approach based on the oracle property of the Lasso (e.g., Fan and Li, 2001; Zou, 2006). When the implementation of the LASSO is an oracle procedure, it will select the correct sparsity structure for the model and will possess the optimal estimation rate. Accordingly, in a first step we could use a Lasso to select the relevant model (i.e. the subset of relevant x) and in a second step we could obtain parameter estimates after conditioning on this subset. We term this procedure Post-Lasso-MLE and use it as a benchmark in later sections. In practice, the post-Lasso-MLE is a straw-man since several authors have already found poor small-sample properties for such post-regularization estimators (e.g. Leeb and Potscher, 2008) that, effectively, ignore the model uncertainty by placing a degenerate prior with infinite mass on the model selected by the first stage Lasso.

4 Personalized Pricing at Ziprecruiter.com

We analyze personalized pricing empirically through a sequence of experiments in collaboration with Ziprecruiter.com. The first experiment uses a sample of prospective, new Ziprecruiter consumers to train a demand model with heterogeneous price responses. The second experiment uses a new sample of prospective consumers to validate the predictions of the model and performance of the personalized pricing structure out of sample. Of interest is whether a firm, like Ziprecruiter, could in fact generate sufficient incremental profits to want to pursue a databased price discrimination strategy. Moreover, we want to analyze the implications for consumer welfare.

Ziprecruiter.com is an online firm that specializes in matching jobseekers to potential employers.

We focus on Ziprecruiter's business-to-business decision since they offer their jobseeker services for free and only charge prospective employers. Hereafter, we refer to prospective employers who could use Ziprecruiter's service as consumers. The firm caters to a variety of potential consumers across various industries that can use Ziprecruiter.com to access a stream of resumes of matched and qualified candidates for recruiting purposes. Customers pay a monthly subscription rate that they can cancel at any time. In a typical month in 2015, Ziprecruiter hosted job postings for over 40,000 registered paying consumers. During the late spring of 2015, Ziprecruiter was in the process of re-evaluating its pricing policy, making them open to our proposal to run randomized field experiments to measure demand and market power.

Our analysis focuses on prospective consumers who have reached the paywall at Ziprecruiter.com for the first time. Amongst all prospective consumers, Ziprecruiter's largest segment consists of the "starters," small firms with typically less than 50 employees, looking to fill between 1 and 3 jobs. Since starters represent nearly 50% of the consumer base, we focus our attention on prospective starter firms. Another advantage of focusing on small consumers is that they are unlikely to create externalities on the two-sided platform that would warrant lower pricing. For instance, Ziprecruiter might want to target low prices to certain very large recruiters in spite of high willingness-to-pay to create indirect network effects that stimulate demand from the set of applicants submitting their resumes. At the beginning of this project the base rate for a "starter" firm looking for candidates was $99/month.

Each prospective new firm that registers for Ziprecruiter's services navigates a series of pages on the Ziprecruiter.com website until they reach the paywall. At the paywall, they must use a credit card to pay the subscription fee. Immediately before the request for credit card information, a consumer is required to input details regarding the type of jobs they wish to fill as well as characteristics describing the firm itself. During this registration process, the consumer reports several characteristics of its business and the specific job posting. Table 3 summarizes the variables we retained for our analysis from the much larger set of registration features[15]. While the set looks small, it generates 133 variables[16]. After completing this registration process, the consumer reaches a paywall and receives a price quote. The registration process is used to ensure that Ziprecruiter's matching algorithm connects consumers with the most relevant CV's of potential applicants. In this case, we believe that the self-reported information is incentive compatible and that we do not need to worry whether consumers strategically mis-report.

[15]In our personalized pricing application below, we only analyze segmentation schemes based on these features which are voluntarily and knowingly self-reported by consumers. We do not use any involuntary information tracked, for instance, through cookies.

[16]An initial set of marginal regressions were used to select these variables from the broader set of thousands of features for the demand analysis (e.g., Fan, Feng, and Song, 2012). For our analysis here we take these selected variables as given.

4.1 Empirical Model of Demand

Assume that a prospective, new consumer i with observable features x_i obtains the following incremental utility from purchasing versus not purchasing

$$\begin{aligned}\Delta U_i &= \alpha_i + \beta_i p_i + \varepsilon_i \\ &= \alpha\left(x_i;\theta_\alpha\right) + \beta\left(x_i;\theta_\beta\right) p_i + \varepsilon_i\end{aligned} \tag{10}$$

where $\alpha\left(x_i;\theta_\alpha\right)$ is an intercept and $\beta\left(x_i;\theta_\beta\right)$ is a slope associated with the price, p_i. To conform with our notation in section 2, we re-write equation 10 as follows

$$\Delta U_i = \tilde{p}_i' \Psi_i + \epsilon_i \tag{11}$$

where $\Psi_i = \left(\alpha\left(x_i;\theta_\alpha\right), \beta\left(x_i;\theta_\beta\right)\right)'$ and $\tilde{p}_i = \left(1\ p_i\right)'$.

The probability that consumer i buys a month of service at price $p_i$is

$$\begin{aligned}\mathbb{P}\left(y_i = 1|p_i;\Psi_i\right) &= \int 1\left(\Delta U_i > 0\right) dF_\epsilon\left(\varepsilon_i\right) \\ &= 1 - F_\epsilon\left(-\tilde{p}_i'\Psi_i\right)\end{aligned}$$

where $y_i = 1$ if she purchases or 0 otherwise.

For our analysis below, we use a linear specification of the functions α and β

$$\begin{aligned}\alpha\left(x_i;\theta_\alpha\right) &= x_i'\theta_\alpha \\ \beta\left(x_i;\theta_\beta\right) &= x_i'\theta_\beta.\end{aligned}$$

We also assume that the random utility disturbance ϵ_i is distributed i.i.d. logistic with scale parameter 1 and location parameter 0. These assumptions give rise to the standard binary Logit choice probability

$$\mathbb{P}\left(y_i = 1|p_i;\Psi_i\right) = \frac{\exp\left(\tilde{p}_i'\Psi_i\right)}{1+\exp\left(\tilde{p}_i'\Psi_i\right)}. \tag{12}$$

Note that our demand specification assigns a continuous treatment effect to prices since, one of our objectives will consist of optimizing prices, on the supply side. This smooth and continuous price treatment effect is an important distinction from most applications of machine learning which involve categorical treatment variables.

4.2 Experiment One: Demand, Pricing and Consumer Welfare

The first experiment was conducted between August 28, 2015 and September 29, 2015. During this period, 7,867 unique prospective consumers reached Ziprecruiter's paywall. Each prospective consumer was randomly assigned to one of ten experimental pricing cells. The control cell consisted of Ziprecruiter's standard $99 per month price, row one of Table 2. To construct our test cells, we changed the monthly rate by some percentage amount relative to the control cell. Following Ziprecruiter's practices, we then rounded up each rate to the nearest $9. The nine test cells are summarized in rows two to ten of Table 2.

4.2.1 Model-free analysis

We report the results from the first experiment in Figure 1. As expected, we observe a statistically significant, monotonically downward-sloping pattern of demand. Demand is considerably less price elastic than Ziprecruiter's current pricing would imply. A 100% increase in the price from $99 to $199 generates only a 25% decline in conversions. Given that most of Ziprecruiter's services are automated and it currently has enough capacity to increase its current consumer base by an arbitrary amount, the marginal cost per consumer is close to $0. Therefore Ziprecruiter is likely under-pricing its service, at least under myopic pricing that optimizes current monthly profits.

Figure 2 plots Ziprecruiter's expected monthly revenue per consumer at each of the tested prices. The plot reveals a considerable degree of unexercised market power, suggesting that Ziprecruiter is significantly under-pricing. . Along our grid of tested price levels, the average monthly revenue per prospective consumer is maximized at $399. Although, once we take into account statistical uncertainty, we cannot rule out that the revenue-maximizing price lies somewhere between $249 and $399, or even above $399.

The static profit analysis does not account for the fact that raising the monthly price today not only lowers current conversion, it may also lower longer-term retention in ways that impact long-term profitability. Figure 3 reports the expected net present value of revenues per consumer over the 4-month horizon from September to December, 2015. The top panel assumes a discount factor of $\delta = 0$ and, therefore, repeats the static expected revenues discussed above. The bottom panel assumes a discount factor of $\delta = 0.996$, implying a monthly interest rate of 0.4% (or an annual interest rate of 5%). While the net present value of profits is much higher at each of the tested prices, our ranking of prices is quite similar. To understand this finding, table 5 reports both the acquisition rate (from September) and the retention rate (for October to December) for each of the tested price levels. As expected, conversion and retention both fall in the higher-price cells. However, survival rates are still low enough that the profit implications in the first month overwhelm the expected future profits from surviving consumers. In sum, our relative ranking of prices does not change much if we consider a longer-term planning horizon.[17] In fact, one month after the experiment, Ziprecruiter increased its price to $249 per month and has retained this base price until at least as recently as May 2021.

[17]Our discussion here assumes that all customers who churn out of Ziprecruiter's business will never return. In practice, consumers may have heterogeneous reasons for churning out including ranging from the satiation of their current recruiting needs to dissatisfaction with the service.

Although not the main focus of our studies, even in the absence of consumer information, purchase and price data alone reveal unexercised market power in this case study. Ziprecruiter should raise its prices by more than 100%, which would generate substantial incremental revenues per consumer. A price increase mechanically reduces consumer surplus; however Ziprecruiter would have eventually learned its demand and raised its price as predicted by any standard microeconomics textbook. The determination of the exact optimal uniform price and the personalized pricing structure requires us to estimate the proposed demand model. In the next section, we discuss the demand estimates.

4.2.2 Demand estimation

We now use the data from the field experiment to estimate the Logit demand model using our WLB estimator discussed in section 3.2[18]. Since the experiment randomized the prices charged to each consumer, we do not face the usual price endogeneity concerns associated with demand estimation using observational databases (e.g. Berry, 1994).

Our demand specification allows for a heterogeneous treatment effect of the price on demand. To accommodate heterogeneity, we use 12 categorical feature variables that are self-reported by the prospective consumers during the registration stage. We break the different levels of these variables into 133 dummy variables, summarized in the vector x_i. We include the main effects of these 133 dummy variables in the intercepts of our model, α, and the 133 interaction effects with price in the slope, β.[19]

In addition to our WLB estimates, we also report results from other approaches that are easier to implement than WLB. We report the MLE estimates of a model that includes all 266 covariates (main effects and interaction effects with price), which we expect would suffer from over-fitting. MLE is much easier to estimate computationally, but faces potential over-fitting problems. In addition, we report results from the unweighted Lasso penalized regression estimates with optimal penalty selected by cross-validation. While the Lasso is easier to implement than WLB, it has the disadvantage of not allowing us to characterize statistical uncertainty and conduct inference. For both the Lasso and the WLB, we always retain the main effect of price. However, even when we do not force price to be retained, the main price effect is always found to be part of the active set.

To compare these specifications, Table 4 reports in-sample and out-of-sample fit measures. We assess model fit using the Bayesian Information Criterion (BIC), the asymptotic approximation of

[18] We use the gamlr function in the R package "gamlr" to implement the logistic Lasso at each iteration of our Bayesian Bootstrap. We simulate the weighted Lasso procedure as follows. For each iteration, we draw a vector of weights for each observation in our sample. We then draw a subsample by drawing with replacement from the original sample using our weights. The logistic Lasso is then applied to this new subsample.

[19] The methods proposed herein scale well with larger sets - we have implemented a version for the firm with the complete set of covariates. Others have had success with the general approach. For instance, Taddy (2015a) successfully implements the approach in a distributed computing environment for applications with thousands of potential covariates.

the Bayes factor which can be used to select between models based on their posterior probabilities (Schwarz, 1978). Since the BIC includes a penalty for the number of parameters, it is robust to over-fitting concerns. For MLE, we report the BIC. For Lasso, the BIC includes a penalty for the number of model parameters (e.g., Zou, Hastie, and Tibshirani, 2007). For our WLB estimator, we report the range of BIC values across the 100 bootstrap replications of the Lasso estimator used for constructing our Bayesian Bootstrap estimate of the posterior, $F(\Theta)$.

We evaluate in-sample fit using the entire sample. As expected, Table 4 shows that the switch from MLE to Lasso improves the in-sample BIC: 10,018 versus 8,366. This improvement is consistent with our concern that the MLE using all the features will over-fit the data. Recall that our objective with the WLB is not prediction, but rather inference. The fact that WLB provides comparable fit to the Lasso in-sample, with an average BIC (across bootstrap replications) very similar to the Lasso's BIC, indicates that we have not sacrificed predictive power in the process.

Our results suggest that regularization matters quite a bit which speaks to the importance of variable selection and model uncertainty. Across the 100 bootstrap replications we conduct, models retain as few as 58 to as many as 188 features in the "active set," the variables included in the model. 172 of the features have more than a 50% posterior probability of being non-zero (i.e., are retained in over 50% of the bootstrap replications). If we look at the 6 parameters with more than 90% posterior probability of being non-zero, these include diverse factors such as "job in British Columbia", "company type: staffing agency," "employment type: full_time" and "is resume required." The fact that we do not see a systematic type of variable exhibiting high posterior probability reinforces the importance of using regularization to select model features as opposed to selecting features manually based on managerial judgment.

As an additional verification, we also examine the out-of-sample predictive fit of each of our estimators in the second column of Table 4. We first split the sample into training and prediction sub-samples, randomly assigning 90% of the consumers to the training sample and the remaining 10% to the prediction sample. We run each specification using the training sample. We report the out-of-sample RMSE and hit rate to assess model prediction. The hit rate classifies each respondent as choosing the alternative with the highest predicted probability. The WLB slightly out-performs both alternative models on RMSE. While it also generates better out-of-sample choice predictions than MLE, it provides identical choice predictions to the basic LASSO. This latter result is not altogether surprising and merely highlights the importance of regularization for our high-dimensional feature set. The key advantage of WLB lies in its ability to generate reliable inferences, as demonstrate below in section 4.5, where we use a second field experiment to assess the sampling properties of the three estimators.

4.3 Decision-Theoretic Pricing

We now use our WLB demand estimates to calibrate Ziprecruiter's decision-theoretic price optimization problems. Since we do not impose any restrictions on the range of parameter values, we cannot rule out the possibility of positive price coefficients or excessively large willingness-to-pay, two issues that could interfere with the optimization. For the price optimization procedures, we top-coded any draws for which $\mathbb{E}\left[\beta\left(x_i\right)|D, x_i\right] \geq 0$ at the highest negative value of $\mathbb{E}\left[\beta\left(x_i\right)|\mathbf{D}\right]$ [20]. In section 6 below, we explore the sensitivity of our results to a more sophisticated deep learning algorithm. All of the price coefficients are found to be negative under the deep learning algorithm. We also show that our main pricing-related findings based on the lasso are robust to the deep learning algorithm.

Table 6 summarizes the predicted economic outcomes associated with the different price structures considered. For each pricing structure, we report the corresponding posterior expected conversion rate (i.e. share of consumers that pay for a month of service), posterior expected revenue per consumer and posterior expected consumer surplus. 95% posterior credibility intervals are also reported for each of these predicted outcomes.

We begin with an analysis of optimal uniform pricing. At Ziprecruiter's base price of \$99, the posterior expected own-price elasticity of demand is only -0.33 with a 95% posterior credibility interval of (-0.41,-0.26). Consistent with our model-free analysis above, Ziprecruiter.com was pricing on the inelastic region of demand prior to the experiment. Recall from Figure 2 that the revenue-maximizing price appeared to lie between \$249 and \$399. The posterior expected own-price elasticity is -0.82 for a price of \$249, and -1.15 for a price of \$399.

The decision-theoretic optimal uniform price, as defined in equation 5, is \$327. Comparing column 3 of the first and second rows of Table 6, we can see that the optimized uniform pricing policy increases Ziprecruiter's posterior expected revenue per consumer by over 55% relative to its \$99 base price, in spite of lowering conversion from 25% to 12%. Not surprisingly, we find an approximately 100% posterior probability that uniform optimal pricing is more profitable than \$99.

We can use our demand estimates to conduct another check that Ziprecruiter would indeed optimally increase its price relative to \$99, even after accounting for the discounted future cash flows from retained consumers. Assume that the a consumer's retention probability in any given month is identical to the acquisition probability. The uniform optimal price that maximizes discounted cash flows is then:

$$p^{NPV} = \underset{p}{\operatorname{argmax}} \frac{1}{1-\delta \mathbb{P}\left(y_i=1|p\right)} p \mathbb{P}\left(y_i=1|p\right) \tag{13}$$

where δ is the discount factor. If we assume $\delta = 0.996$, we obtain $p^{NPV} = \$261$ which once again

[20] This top-coding only affects 6% of the posterior draws of $\left\{\beta^b\left(x_i\right)\right\}_{b=1}^{B}$.

confirms the sub-optimality of the \$99 price.

We now explore decision-theoretic personalized pricing. Figure 4 summarizes the degree of estimated heterogeneity across consumers. In panel (a), we report the distribution of consumers' posterior mean price sensitivities

$$\mathbb{E}\left[\beta\left(x_i\right)|\mathbf{D}, x_i\right] \quad = \quad \frac{1}{B}\sum_{b=1}^{B}\beta^b\left(x_i\right).$$

The dispersion across consumers suggests a potential opportunity for Ziprecruiter to price discriminate. In panel (b), we report the distribution of posterior mean surplus across consumers when Ziprecruiter prices its monthly service at \$99:

$$\mathbb{E}\left[V\left(p,x\right)|\mathbf{D}, x_i, p=\$99\right] = -\frac{1}{B}\sum_{b=1}^{B}\frac{\log\left(1+\exp\left(\alpha^b\left(x_i\right)-\$99\times\beta^b\left(x_i\right)\right)\right)}{\beta^b\left(x_i\right)}. \tag{14}$$

Panel (b) illustrates the wide dispersion in dollar value consumers derive from the availability of Ziprecruiter when it costs \$99. The 2.5^{th} percentile, median and 97.5^{th} percentile willingness-to-pay are \$23.55, \$99.04 and \$443.59 respectively. The magnitudes and degree of dispersion in value indicate an opportunity for Ziprecruiter to price discriminate using the registration features as a segmentation scheme.

We find considerable dispersion in the prices, ranging from as low as \$126 to as high as \$6,292. Across our $N = 7,866$ consumers, all of the personalized prices are strictly larger than Ziprecruiter's \$99 baseline price. In spite of the range of prices, some exceeding \$1,000, the median price is \$277, which is much lower than the optimal uniform price, \$327. Therefore, the majority of consumers would benefit from personalized pricing relative to uniform pricing. Comparing column 3 of the second and third rows of Table 6, we see that the decision-theoretic personalized pricing increases Ziprecruiter's posterior expected revenue per consumer by 19% relative to uniform pricing, from \$39.01 to \$46.57. Moreover, compared to Ziprecruiter's base price of \$99, decision-theoretic personalized pricing increases posterior expected revenue per consumer by 86%.

A concern with our personalization scenario is that about one quarter of our recommended prices exceed the highest price in the experiment, \$399, with many in excess of \$1,000. Ziprecruiter's management team indicated that they would be unlikely to consider prices above \$499[21]. In the fourth row of Table 6, we re-compute the decision-theoretic prices when we impose an upper bound of \$499. As expected, this cap increases the posterior expected conversion to 13%. Expected posterior revenue per consumer is still 8% higher than under uniform pricing. The expected posterior revenue per consumer from capped personalized pricing exceeds that of uniform pricing with a posterior probability of 98%.

[21] This cap reflected both concerns with projecting too far outside the range of the data and, more importantly, charging prices that they felt might create negative goodwill with consumers.

Based on conversations with Ziprecruiter management, we also do not expect any competitive response from other platforms. Our recommendations involve increasing (not decreasing) prices above the baseline of $99, mitigating any concerns about triggering a price war.

The incremental profitability of personalization in general depends crucially on the "no arbitrage" condition which rules out unintended strategic behavior by consumers (e.g., (Fudenberg and Villas-Boas, 2006; Chen, Li, and Sun, 2015; Bonatti and Cisternas, 2018)). In the Ziprecruiter context, the "no arbitrage" condition requires that consumers self-report their company features truthfully during the registration stage. There is no way for us to verify the accuracy of the self-reported features. However, we showed above that company features predict demand and, in section 4.5 below, we show that personalization generates higher profits out of sample than alternative pricing structures (e.g., uniform optimal pricing) that do not rely on self-reported features. We also believe that truthful self-reporting will remain incentive compatible at Ziprecruiter in the longer-term for at least three reasons. First, most consumers would not learn about differential pricing because Ziprecruiter does not post its prices in a public manner. A firm must complete the registration process to obtain a price quote, making it difficult to use software to scrape Ziprecruiter's prices under different registration profile responses. Second, consumers face an arbitrage cost in the sense that mis-reporting features has an adverse effect on Ziprecruiter's key service: the resume-matching algorithm uses company features to determine the ideal recruiting prospects. Arbitrage costs are prevalent in other industries that have studied personalized pricing. For instance, in the consumer packaged goods industry, consumer transaction histories are used to determine differential price elasticities (e.g., Rossi, McCulloch, and Allenby, 1996; Chintagunta, Dubé, and Goh, 2005)). A high willingness-to-pay consumer would need to purchase less preferred brands on a regular basis in order for her purchase history to generate a high price-elasticity signal.[22] Third, it would not be possible for a consumer to determine which combination of features generates low prices purely because of the complexity of the WLB algorithm that uses 133 features. Nevertheless, we cannot rule out challenges with the no-arbitrage condition in the longer-term at Ziprecruiter or for other industries with lower transaction costs for price discovery (e.g., without a registration requirement).

4.4 The Information Content of Features

We now explore the types of consumers that benefit from personalized pricing. While our experiment was not designed to recover the causal effect of specific individual firm features on willingness-to-pay, it is nevertheless interesting to analyze the role of feature information as an exploratory exercise. We find that the job benefit features are the most highly correlated with the personalized prices. For instance, "job total benefits" and the presence of "medical benefits"

[22] Arbitrage costs also arise in the emerging trend of geographic targeting using mobile coupons. High willingness-to-pay consumers would need to incur time and travel costs to visit and dwell in locations associated with lower willingness-to-pay in order to receive a discount (e.g., Dubé, Fang, Fong, and Luo, 2017)).

have a correlation of 0.31 and 0.27, respectively, with the personalized price levels. However, the correlational value of information can be clouded by the fact that certain features, such as state and company type, comprise many underlying dummy variables (e.g., 62 state/province dummy variables) that may be important drivers of prices collectively.

As an exploratory exercise, we classify each of the feature variables into $g = 1, ..., 6$ groups: state, benefits, job category, employment type, company type and declared number of job slots. We then use entropy to measure the incremental information content associated with a feature group. Let $\mathcal{X}$ represent the complete feature set and let $f\left(p^*|\mathcal{X}\right)$ denote the density of personalized prices based on information set $\mathcal{X}$. To assess the targetable information in each group g, we drop all of its corresponding features and rerun the WLB algorithm and the personalized pricing calculations to derive $f\left(p^*|\mathcal{X}_{-g}\right)$ where $-g$ denotes the exclusion of feature group g. We then compute the Kullback-Leibler divergence in the distribution of personalized prices when we exclude feature group g:

$$KLD\left(\mathcal{X}||\mathcal{X}_{-g}\right) = \int_p f\left(p|\mathcal{X}\right) \log\left(\frac{f\left(p|\mathcal{X}\right)}{f\left(p|\mathcal{X}_{-g}\right)}\right).$$

We effectively treat $f\left(p^*|\mathcal{X}_g\right)$ as our target distribution so that $KLD\left(\mathcal{X}||\mathcal{X}_{-g}\right)$ measures the entropy associated with approximating $f\left(p^*|\mathcal{X}\right)$ using $f\left(p^*|\mathcal{X}_{-g}\right)$, the distribution of prices based on the narrower information set that excludes the feature group g.

We can now assess the relative incremental information associated with each feature group by ranking them in terms of divergence. State is the most informative group ($KLD\left(\mathcal{X}||\mathcal{X}_{\text{-}\{state\}}\right) = 0.032$), followed by job category ($KLD\left(\mathcal{X}||\mathcal{X}_{\text{-}\{job\ category\}}\right) = 0.029$), benefits ($KLD\left(\mathcal{X}||\mathcal{X}_{-\{\text{benefits}\}}\right) = 0.018$), employment type ($KLD\left(\mathcal{X}||\mathcal{X}_{-\{\text{employment type}\}}\right) = 0.0078$), company type ($KLD\left(D||D_{-\{\text{company type}\}}\right)$ 0.004) and declared number of job slots ($KLD\left(D||D_{-\{\text{job slots}\}}\right) = 0.002$). Since company type and state each require only a single categorical question during the registration process on Ziprecruiter's website, these information sources are more efficient to elicit from prospective consumers. In sum, individual features like company size and benefits are the most correlated with personalized prices. However, aggregating information into groups, the distribution of personalized prices seems most influenced by broad job categories and geographic locations.

4.5 Experiment Two: Validation

A novel feature of our study is that we conducted a second field experiment to test the policy recommendations based on our empirical analysis of the first experiment. This second experiment allows us to confirm the predictive validity of our structural analysis in the previous section.

We conducted the second field experiment between October 27, 2015 and November 17, 2015 using a new sample of prospective consumers that arrived to the Ziprecruiter.com paywall during this period and had not previously paid for the firm's services. Each prospective consumer was randomly assigned to one of the three following pricing structures:

1. Control pricing – \$99 (25%)
2. Uniform pricing – \$249 (25%)
3. Personalized pricing (50%).

We over-sampled the personalized pricing cell to obtain more precision given the dispersion in prices charged across consumers.

The tested pricing structures were formulated in part based on Ziprecruiter's own needs. For instance, as we explained earlier, they chose a uniform price of \$249 because, based on the earlier experiment, (i) the profit implications relative to the optimum were minimal and (ii) the management believed that \$249 would be more palatable on account of similar prices being used elsewhere in the industry. For our personalized pricing cell, consumers were charged a price based on the values of x_i they reported during the registration stage. As we indicated in the previous section, Ziprecruiter capped the personalized prices at \$499. In addition, they asked us to round the personalized price down to the nearest \$9, discretizing the prices into \$10 buckets ranging from \$119 to \$499. For instance, a consumer with a targeted price of \$183 would be charged \$179. Ziprecruiter used this rounding because they believed consumers would find the \$9 endings on prices more natural. Based on our demand estimates, this rounding has very little impact on the predicted profits of personalization.

During this period, 12,381 prospective consumers reached Ziprecruiter's paywall. Of these prospectives, 5,315 were starters and the remainder were larger firms. Amongst our starters in the November 2015 study, 26% were assigned to control pricing, 27% to the uniform pricing and 47% to the personalized pricing. In the personalized pricing cell, the lowest price was \$99 and, hence, neither of our test cells ever charged a prospective consumer less than the baseline price of \$99.

To verify that our three experimental cells are balanced, we compare the personalized prices that would have been used had we implemented our personalized pricing method in each cell. Figure 5 reports the density of personalized prices in each cell. For the control cell (\$99) and test cell (\$249), these are the personalized prices that subjects would have been shown had they been assigned to the personalized pricing test cell instead. The three densities are qualitatively similar, indicating that the nature of heterogeneity and willingness-to-pay is comparable in each cell. This comparison provides a compelling test for the balance of our randomization as it indicates that our distribution of personalized prices would look the same across each of the experimental cells.

4.5.1 Out-of-Sample Validation of Model Predictions

A novel feature of our case study is the ability to use the November 2015 experiment to validate our proposed WLB inference procedure along with the predictions from our structural model and the corresponding inferences regarding profits under different pricing structures discussed in section 4.3. The box plots in Figure 6 compare the realized sampling distribution for conversion across several of the tested price cells to the corresponding inferences for conversion using our WLB approach versus the the post-Lasso MLE and classical MLE approaches (as discussed at the end of section 3.2). To account for sampling error in our realized outcomes, we bootstrap our sample $1,000$ times (sampling with replacement). For WLB, we use the draws from the posterior distribution. For post-Lasso MLE and MLE, we use a parametric bootstrap from the asymptotic covariance matrix. The box plots indicate that WLB comes much closer to approximating the observed sampling distribution in conversion rates across price cells. Relative to WLB, both post-Lasso MLE and MLE generate what appear to be strikingly under-stated degrees of statistical uncertainty. This is not surprising since, unlike post-Lasso MLE, WLB accounts for model uncertainty. Unlike MLE, WLB uses regularization to avoid model over-fitting. At the bottom of each panel, we report the Kullbach-Leibler divergence for each of our three estimators relative to the true distribution of realized conversions. The divergence of WLB is always considerably smaller than for post-Lasso MLE and MLE, often by orders of magnitude. These findings suggest that WLB is providing a reasonable approximation of the posterior uncertainty over both the model specification and feature weights. The results also suggest that personalized pricing for a company like Ziprecruiter is a Big Data problem in the sense that the selection of model features plays an important role in addition to the usual estimation of feature weights.

In Table 7, we report the realized conversion rates and revenue per consumer across our three pricing structures, control ($99), test ($249) and test (personalized pricing). For realized outcomes, we report the 95% confidence interval. We also report the posterior expected conversion rate and revenue per consumer in each of the three cells based on our estimates from the September 2015 training sample. Specifically, we use the posterior distribution of the parameter estimates, $F\left(\Theta|\mathbf{D}^{Sept}\right)$ and the observed features from our November subjects, X^{Nov}, to form our predictions. For each posterior mean, we also report the corresponding 95% credibility interval.

Starting with the realized outcomes, average conversion is higher in the control cell which has the lowest monthly price, as expected. Average conversion is almost identical in the uniform and personalized pricing cells, at 15%. However, the average profit per consumer is higher in the personalized pricing cell, as one would theoretically expect. Overall, the uniform pricing increases expected profits per consumer by 67.74% relative to control pricing; although our bootstrapped confidence interval admits a change as low as 46%. Personalized pricing increases expected profits by 84.4% relative to control pricing; although our bootstrapped confidence interval admits a change as low as 64%. These improvements from price discrimination are consistent with our predictions based on the September sample discussed above in section 4.3. Finally, although not reported, our bootstrap generates an 87% probability that personalized pricing profits will exceed uniform profits.

These realized conversion rates and revenues per consumer are broadly consistent with our model predictions. In particular, the predicted outcomes for the uniform pricing at $249 and the personalized pricing are almost identical to the realized values. These findings provide out-of-sample validation of the predictive value of our WLB estimator and our structural demand model. The second experiment also allows us to test our pricing policies out of sample. A test of the hypothesis that uniform pricing at \$249 is more profitable than uniform pricing at \$99 is strongly significant ($p<0.01$). A test of the hypothesis that personalized pricing is more profitable than uniform pricing at \$249 is less precise ($p=.069$), although the point estimates for both cells correspond closely with our Bayesian predictions.

5 Personalization, Data Policies and Consumer Welfare

Having established that personalized pricing (large-scale third-degree price discrimination) generates a substantial increase in producer surplus, we now turn to the demand side of Ziprecruiter's business-to-business market. As explained earlier, Ziprecruiter was in the process of exploring ways to collect demand data and improve its pricing when we began the collaboration. Therefore, we use the optimal uniform price as our base case, not \$99, since the former reflects the textbook inverse-elasticity-rule pricing that would be predicted for a maturing company. Our analysis also focuses on the role of conditioning on features x_i to set prices. One could implement uniform pricing with a demand model that does not condition on X_i for estimation, instead using only price and conversion data. Although not reported herein, the optimal uniform price is almost identical in that case (i.e., \$324 as opposed to \$327). So for the remainder of our analysis, each of our pricing structures uses the same demand estimates, $\mathbb{P}\left(y_i = 1 | p; x_i\right)$.

In what follows we examine two aspects of consumer welfare (i) The aggregate welfare differential created by the change in pricing policy and (ii) the impact of data policies on consumer surplus.

5.1 Consumer Welfare

To analyze the consumer welfare implications of personalized pricing relative to optimal uniform pricing, recall that we use Atkinson (1970)'s mean of order r class of consumer welfare functions, which in our empirical setting corresponds to:

$$S_r\left(\mathbf{p}\right) = \left[\frac{1}{N}\sum \mathbb{E}\left(V\left(\mathbf{p}, x_i\right)\right)^r\right]^{1/r} \tag{15}$$

where $V\left(\mathbf{p}, x_i\right)$ corresponds to the individual-level surplus as in equation (14). As discussed earlier, we follow Lewbel and Pendakur (2017) and restrict our attention to $r \in \{-1, 0, 1\}$ corresponding to the harmonic, geometric and arithmetic means the first two of which reflect inequality averse preferences (on the part of the planner).

Panel (a) of Table 8 reports our consumer welfare results for each decision-theoretic pricing structure. We start with row three, corresponding to the conventional "total consumer surplus" standard, $r = 1$, for which the welfare function (15) is inequality neutral. Personalization reduces linearly aggregated consumer surplus considerably relative to uniform optimal pricing from \$94.78 to \$71.41 (25%), and by more than the increase in profits. Given the decline in conversion under personalized pricing, it is not surprising that we observe a decline in total surplus (firm and consumer). This decline in total surplus comes from less than half the consumers. In fact, 63% of the consumers' personalized prices are lower than the uniform optimal price of \$327, indicating that over half our consumers benefit from personalization even though total surplus is lower. Since all consumers are weighted equally, a small number of consumers exert an inordinate amount of influence on the the average. We now turn to the inequality-averse consumer welfare functions, $r = -1$ and $r = 0$, respectively, and report the corresponding results in the first and second rows of Table 8 panel (a). Under both inequality-averse consumer welfare functions, personalization is preferred because the allocative benefits outweigh the decline in total surplus. These findings indicate how the articulation of the aggregate consumer welfare effects of a change in pricing policy depends on the planner's preferences and the choice of surplus aggregation metric. Although not reported in the Table, we find that welfare is equal under uniform and personalized pricing at $r = 0.06$, suggesting that some amount of inequality-aversion is required for social welfare to improve under personalized pricing.

For completeness, Panels (b) and (c) of Table 8 provide welfare calculations for two other comparisons: "implemented personalized vs optimal uniform" and "implemented personalized vs implemented uniform," respectively. The term "implemented" refers to the \$499 personalization cap and the uniform price of \$249 that were implemented by Ziprecruiter in practice. As one would expect, introducing a price cap at \$499 increases total consumer surplus considerably (under all metrics). It follows then that personalization is viewed more favorably than in panel (a). In particular, linearly aggregated consumer surplus falls by 2.04% (in contrast to 25% in panel(a)) relative to uniform pricing, while still allowing the firm to generate a more than 8% gain in profits. We do not claim that the use of such caps and the results herein would generalize to other firms and/or other industries where personalized pricing could be implemented. Finally, panel (c) of Table 8 shows that when comparing the implemented versions of personalized and uniform pricing, personalization is preferred only under the more extreme inequality-averse welfare function (harmonic mean with $r = -1$). The shift towards uniform pricing reflects the fact that Ziprecruiter implemented a much lower price than optimal (\$249 vs \$327) by selecting a value off the test grid instead of maximizing its posterior expected profits. In this case, the lower uniform price more than offsets the benefits of a more equitable allocation of surplus unless inequality-aversion is strong.

5.2 Data Policies and Consumer Surplus

Policies like GDPR and CCPA have been enacted to protect consumer's privacy broadly, but also to prevent firms from surplus extraction. Theoretically, however, it is possible that restricting the types of data firms are permitted to use for personalized pricing could harm consumer surplus (e.g., Bergemann, Brooks, and Morris, 2015). We will now use our Ziprecruiter case study to explore how restrictions over the set of features available to a firm for pricing purposes affects consumer surplus. For most of of the analysis that follows, we focus on the usual aggregate surplus metric with linear aggregation (i.e., $r = 1$).[23]

Formally, we need to recast the analysis in Bergemann, Brooks, and Morris (2015) for the context of data-based marketing. Suppose the firm uses all the available data to estimate the demand parameters, $F_\Theta(\Theta|\mathbf{D})$, as before. However, suppose also that the firm is only permitted to use a subset of the $g = 1, ..., 6$ sets of consumer features for the personalization of prices. Let $\mathcal{X}$ represent the complete feature set, let $\mathcal{X}^o \subset X$ denote the subset of features the firm can use for segmenting consumers and setting personalized prices, and let $\mathcal{X}^u \subset X$ represent the features the firm cannot use for segmentation. The firm can partition demand for a consumer i with features X_i into the targetable and non-targetable components as follows:

$$\mathbb{P}(p; X_i^o, \Theta) = \frac{1}{1 + \exp\left(-\left(\alpha\left(X_i^o, X_i^u, \Theta\right) + \beta\left(X_i^o, X_i^u, \Theta\right)p\right)\right)}$$

where

$$\alpha\left(X_i^o, X_i^u, \Theta\right) = \alpha + X_i^o\alpha_o + X_i^u\alpha_u$$
$$\beta\left(X_i^o, X_i^u, \Theta\right) = \alpha + X_i^o\alpha_o + X_i^u\alpha_u$$

For a given segmentation structure, $\mathcal{X}^o$, the personalized pricing problem is

$$p_i^* = \arg\max_p \left\{(p-c)' \int\int \mathbb{P}(p; X_i^o, \Theta)\, dF_{X^u}\left(X^u|X^o\right) dF_\Theta(\Theta|\mathbf{D})\right\} \tag{16}$$

where $F_{X^u}(X^u|X^o)$ represents the firm's beliefs about a consumer's unobserved traits, X^u, conditional on her observed traits, X^o. We use an empirical estimate of $F_{X^u}(X^u|X^o)$ to capture the fact that even though the firm cannot segment on X^u directly, it can nevertheless form an expectation about those unobserved traits from the empirical correlation between features. We solve the personalized prices 16 corresponding to each of the 62 possible combinations of the $g = 1, ..., 6$ feature groups, which includes the case using all the feature variables[24]4.3

[23]Results for $r \in \{0, -1\}$ are available from the authors upon request. In line with our previous discussion it is possible for personalized pricing to be surplus positive relative to uniform pricing is one uses inequality-averse aggregation. The non-monotonicity finding pertaining to data that we discuss below holds even with the alternate aggregation metrics.

[24]We simulate the integrals by using our posterior WLB draws from $F_\Theta(\Theta|\mathbf{D})$ and 100 independent draws from $F_{X^u}(X^u|X^o)$. We use a K-nearest neighbor approach to estimate $F_{X^u}(X^u|X^o)$ using the Hamming distance

We report the range of feasible personalized pricing outcomes in the surplus triangle in Figure 7, the statistical decision-theoretic analog of the feasible surplus allocations examined in Bergemann, Brooks, and Morris (2015). All expectations for posterior surplus are taken over the full posterior distribution, $F_{\Theta}\left(\Theta|\mathbf{D}\right)$. Point A represents the case where the firm has conducted demand estimation, but does not use any of the consumer-level features for segmentation. In this case, the firm charges the optimal uniform price and earns the standard, uniform monopoly profits. Point B represents the purely theoretical case where the firm not only observes all of the consumers' features, it also observes their utility shocks, $\{\epsilon_i\}_{i=1}^{N}$. In this case, the firm conducts perfect price discrimination. See Appendix E for details on the calculation of the expected posterior first-degree price discrimination outcomes (i.e., where the uncertainty is for the analyst and not for the firm). Point C represents the case where consumer surplus is maximized subject to the constraint that the firm earns the expected posterior uniform monopoly profits. Finally, point D represents the case where expected posterior social surplus is minimized, with the firm earning the expected posterior uniform monopoly profits and consumer surplus is zero. Bergemann, Brooks, and Morris (2015) show that every point in this surplus triangle represents a potentially feasible segmentation with third-degree price discrimination.

The top panel of Figure 7 also indicates in blue all of the 62 possible segmentation schemes based on our observed feature set. Point E corresponds to the personalized pricing scenario already discussed and represents the most granular segmentation using all of the observed features. As expected, each of the 62 feasible segmentation schemes is more profitable than uniform pricing. However, these personalized pricing schemes are not nearly as profitable, in expectation, as perfect price discrimination. Even when all the features are used, personalization only generates 30% of the expected posterior profits under perfect price discrimination.

Turning to the demand side, each of our 62 feasible segmentation schemes reduces consumer surplus relative to points C and A (uniform pricing), sometimes by as much as 30% relative to point A. Even though it is theoretically possible for a segmentation scheme to exist that would increase the expected posterior consumer surplus relative to the case of uniform pricing, none of the 62 scenarios achieves this outcome. The best-case scenario, which conditions prices only on the "employment" and "number of declared job slots" features, generates 87% of the consumer surplus under uniform pricing. Recall from above that when we implement Ziprecruiter's price cap at \$499, personalization based on the full feature set reduces consumer surplus by 2% while improving posterior expected profits by over 8%. Our data do not allow us to determine whether firms would implement such price caps in general. We also highlight the point that personalized pricing does come close to the case of true perfect price discrimination, which would extract all the consumer surplus. Even with expanded data collection, it is unlikely that a firm could truly perfectly price discriminate using consumer data. Even in the brand choice literature where

between each of the observations in our training sample and $K = 200$ as our cut-off.

pricing could be conditioned on detailed, individual-level transaction histories, there is still a lot of unpredictable, random brand switches (e.g., Dubé, Hitsch, and Rossi, 2010).

The bottom three panels zoom in on the surplus triangle to examine how different data policies influence consumer surplus. The left-most panel indicates that when we only allow the firm to target prices based on "benefits," total expected posterior consumer surplus is almost \$1 lower than when we also allow the firm to target on "company type" and "declared number of job slots." The removal of these latter two features and using only "benefits" reduces consumer surplus with 87% posterior probability. The middle panel shows a similar result. Only targeting prices on "job category" generates more than \$1 less consumer surplus than when the firm is also permitted to target on "employment type," "company type" and "declared number of job slots." The removal of these latter three features and using only "job category" reduces consumer surplus with 98% posterior probability. However, the right-hand panel indicates that some consumer features strictly harm consumer surplus. In particular, allowing the firm to target on "job category" and/or "state" reduces consumer surplus. These results indicate that allowing the firm to target on more granular data can be good for consumer surplus and that granularity per se does not harm consumers.

In spite of the decline in total consumer surplus, the percentage of consumers that benefit from personalization ranges from 59.4% to 62.2% across our 62 segmentation scenarios. Therefore, less than half the consumers bear the cost of personalization. To see this point more clearly, Figure 8 plots density estimates of the change in posterior expected surplus across consumers for each of the 62 segmentation scenarios versus uniform pricing. In each case, we see a large mass of consumers just to the right of \$0, representing the majority who benefit from personalized prices. We then see a long tail to the left of \$0 representing the minority of consumer who are harmed. If we correlate the incidence that a consumer benefits from personalization ($p_i^* < p^{unif}$) with the consumer features, we find that two most highly correlated features are "Small Company Type" ($corr = 0.38$) and "Part-Time Employment" ($corr = 0.31$). At face value, these results suggests that smaller companies with part-time staff are the most likely to benefit from personalization. In contrast, the most negatively correlated features are all related to job benefits, e.g., "Total Job Benefits" ($corr = -0.81$), "Full-Time Employment" ($corr = -0.37$) and "Medium Company Type" ($\rho = -0.25$).[25] Therefore, larger companies with full-time employment and high benefits are the most likely to be harmed from personalized pricing. Conceptually, this reallocation of consumer surplus from personalized pricing could be rationalized as "fair" under a Pareto-weight scheme that assigns higher social value to smaller, disadvantaged firms.

A key finding from our analysis is that we do not observe a monotonic relationship between the number of features used for segmentation and total consumer surplus or total number of consumers who benefit from personalization. Thus, granting the firm more access to consumer data does not per se lead to more consumer harm. However, this finding must be balanced against the fact

[25] The Large Company Type feature was excluded due to redundancy.

that total consumer surplus falls for each of the segmentation scenarios considered relative to the base case of uniform pricing that does not condition on consumer features. In Figure 7, we can see that the full segmentation using all 6 groups of consumer features generates more consumer surplus than several of the restricted scenarios. For instance, allowing the firm to condition its prices on all 6 feature groups increases consumer surplus by 1.4% relative to restricting the firm to conditioning on “job state,” “benefit” and “company category” (i.e., removing all the features associated with “job benefits,” “number of declared job slots” and “employment type”). Similarly, 61% of the consumers benefit from personalized prices conditioned on all feature variables, whereas only 59.4% benefit when the firm is only allowed to condition its prices on “benefits.” In Figure 8 we see that the density of the change in expected posterior surplus across consumers for full personalized pricing versus uniform pricing is shifted to the right of several of the other restricted segmentation scenarios. In sum, granting the firm access to more information is not per se worse for the consumer as it can lead to segmentation schemes that allocate more surplus to the consumer.

6 Robustness

Our results above used a LASSO regularization algorithm to determine the functional parameters $\{\alpha(x), \beta(x)\}$. In this section, we examine the robustness of our results to a more sophisticated machine learning algorithm to model parameter heterogeneity using the deep learning framework based on Farrell, Liang, and Misra (2021b) and Farrell, Liang, and Misra (2021a).

6.1 A Deep Learning Approach

Unlike the application of ML for prediction purposes, the choice of ML algorithm and the need for model structure are more important in the context of demand estimation and inference. For example, the direct application of a random forest with a standard splitting rule to our demand estimation problem will lead to infinite prices for some subset of consumers. The forest will predict a constant purchase probability for any price at or above $399, the maximum tested price in the experiment. The corresponding revenues will therefore increase without bound in prices and no interior solution will exist. The implementation of shape restrictions on demand to obtain a unique, interior optimal price is difficult for most ML tools and beyond the scope of this paper.

As explained in Farrell, Liang, and Misra (2021a), not all ML methods are “structural compatible” in the sense that they can be embedded directly into a structural parametric model. For example, deep neural networks are are structurally compatible while random forests are not. We now examine the robustness of our results to more flexible deep neural networks that retain the Logit structure of the choice model.

6.1.1 Deep Learning

As before, a consumer with features x facing prices $\tilde{p}_i = (1\ p_i)'$ derives the following incremental utility from buying

$$\begin{aligned} \Delta U_i &= \alpha_i + \beta_i p_i + \varepsilon_i \\ &= \alpha\left(x_i\right) + \beta\left(x_i\right) p_i + \varepsilon_i \end{aligned} \tag{17}$$

and has corresponding choice probability

$$\mathbb{P}\left(y_i = 1 | p_i; \Psi_i\right) = \frac{\exp\left(\tilde{p}_i' \Psi_i\right)}{1 + \exp\left(\tilde{p}_i' \Psi_i\right)}$$

We now model the parameter vector as a deep neural network (DNN)

$$\Psi_i = \left(\alpha\left(x_i\right), \beta\left(x_i\right)\right)' = \Psi_{\text{DNN}}\left(x_i; \theta_{\text{DNN}}\right)$$

Since the observed consumer features in our data are discrete, the advantage of the deep neural net is limited to finding (possibly higher-order) interactions that might be relevant in explaining consumer choices. We use two architectures, one with two hidden layers and another with three layers. In each case, the specification allows for 0 nodes in each layer. We limit the complexity of the model on account of the limited data ($N < 8000$) our application. Our results do not change qualitatively when we perturb the architecture while retaining a comparable degree of complexity of the network. We refer the interested reader to Farrell, Liang, and Misra (2021b) and Farrell, Liang, and Misra (2021a) for a more rigorous discussion of the algorithm and its implementation. As with our Lasso based framework, we implement the Bayesian Bootstrap by optimizing the objective function with randomized Dirichlet weights for $R = 100$ repetitions. This gives us our draws from the approximate posterior $\left(\widehat{\theta}^r_{\text{DNN}}\right)$ and consequently we obtain $\Psi_{\text{DNN}}\left(x_i; \widehat{\theta}^r_{\text{DNN}}\right)$. Our subsequent demand and pricing analysis is analogous to our approach using the WLB above.

6.1.2 Comparison of results using Lasso and Deep Learning

To assess any potential differences between the Lasso and Deep Learning algorithms, we compare the following sets of results: (a) individual parameter estimates, (b) uniform and personalized prices and (c) the differences in consumer welfare across pricing policies.

(a) **Individual parameter estimates:** Figure (10) compares the distribution of the posterior means across consumers for the three methods: Lasso, 2-layer deep learning (DNN-2Layer) and 3-layer deep learning (DNN-3Layer). The panel on the left plots the density of the parameters while the panel on the right displays the box-plots and inter-quartile ranges for each of our algorithms. The dark line in the box plot indicates the median and not the mean. The three distributions are qualitatively similar and cover very similar ranges of the parameter space. The means of the three distributions are quite close, with mean $\beta(x)$ of -0.0058, -0.0054 and -0.006 for the DNN-2Layer, DNN-3Layer and Lasso, respectively.

However, we do observe some noteworthy differences. First, the deep learning based estimators tend to restrict the range of the price coefficient to be negative in spite of the fact we have not imposed any sign restrictions. We conjecture that the potential for interaction effects between features may be leading to a better fit of the price effect. Second, the deep learning parameters imply a higher degree of heterogeneity than those from the lasso. In particular, the price coefficients exhibit higher variance and skewness than their lasso counterpart.

(b) **Pricing Policies:** We obtain qualitatively similar optimal uniform prices under each of our three approaches: \$301.92, \$363.81 and \$323.34 for the DNN-2Layer, DNN-3Layer and Lasso, respectively. To compare personalized prices, we plot the three sets of distributions in Figure (11). While the median personalized prices (as seen in the box-plot) are close, the heterogeneity in these prices is quite different. In particular, the 3-Layer specification exhibits a higher variance corresponding to the higher variance in the parameter estimates. In spite of these differences, the Lasso specification does not show any systematic bias.

(c) **Welfare:** For each of our three approaches, we compare welfare under the uniform and personalized pricing policies. As with the parameters and the optimal prices, our three approaches generate similar differences in welfare under the two pricing policies, including comparable medians (see Figure 12). All three methods find a large difference between the median and the mean (the line with a dot). In all cases, the mean is positive and the median is negative. This difference in sign between the mean and median once again indicates the sensitivity of welfare conclusions to the exact manner in which consumer surplus is aggregated by the social welfare function. Interestingly, the proportion of consumers who are worse off under uniform pricing is higher under the deep learning framework. The intuition here is straightforward: since the price coefficients are well-behaved relative to the Lasso (i.e., fewer values near or greater than zero), the consumer surplus values are less exaggerated under deep learning. Consequently, both the levels of consumer surplus and the differences are less variable in the deep learning framework.

In summary, our key qualitative findings under the Lasso are robust to a more sophisticated deep learning algorithm. All three of our estimators predict that total consumer surplus falls under personalized pricing. However, alternative inequality-averse welfare functions would likely favor personalization over uniform pricing.

6.2 Discussion

In light of the public scrutiny of databased marketing, of interest is how the results from the case study herein affect our beliefs[26] about the welfare implications of personalized pricing (Maniadis,

[26] We thank the editors for suggesting this disussion.

Tufano, and List (2014)). As discussed earlier, the popular press and public policy debate indicates a strong negative prior belief about the impact of personalized pricing on consumer welfare and a strong positive prior about the impact on firm profitability, in spite of the more neutral prior implied by the extant empirical literature. A formal Bayesian update as in Maniadis, Tufano, and List (2014) is infeasible and beyond the scope of this analysis. Having said that, we can use the ideas therein to articulate what the reader might reasonably conclude from our results. As with any Bayesian econometric analysis, the reader's posterior beliefs depends on the evidence, which in this case is a function of the model specification and the data.

Our analysis finds that, on the supply side, personalization increases profitability and, hence, a firm would be likely to implement a personalized pricing structure in a setting like ours, in contrast with recent work that fails to detect incremental profits from discriminating based on observed consumer feature variables. These results are robust to perturbations in data and the methodology used. As such, our analysis strengthens priors beliefs that personalized pricing improves profits.

The demand-side implications are more ambiguous. Our analysis demonstrates that posterior beliefs about consumer welfare is potentially affected by at least three factors: the social planner's preferences over the distribution of consumer surplus (i.e., the welfare function), the amount and nature of data conditioned on for personalized pricing, and the methodology used to analyze the data and classify consumers into "types."

From a methodological perspective, section 6.1.2 shows that our key findings appear to be robust to different machine learning algorithms. Hence, we would conjecture that posterior beliefs about the welfare effects of personalization are not dependent on the ML method.

In contrast, the consumer welfare implications are quite sensitive to the specific welfare function used. Society's exact degree of inequality averion (r) should be part of the reader's subjective prior and any updating of beliefs about personalized pricing will depend crucially on this quantity. For instance, total, linearly aggregated consumer surplus falls. Therefore, under inequality-neutral societal preferences ($r = 1$), our case study supports the a priori concerns expressed in CEA (2015) and would lead to stronger posterior beliefs about the adverse consumer welfare effects of personalized pricing. Under inequality-averse societal preferences ($r \in \{-1, 0\}$) that place some weight on the distribution of surplus across consumers, our case study supports a more favorable posterior belief about personalization due to the allocative effects.

As a concrete example, consider a public policy that might potentially restrict the granularity of data used by the firm for personalized pricing. The posterior belief about the welfare implications of this policy in the context of our case study will be sensitive to the social welfare function adopted. Figure (9) plots the relationship between consumer welfare and the number of features used for personalization for each of the three Atkinson welfare functions, indexed by its respective inequality-aversion parameter r. The dotted line in each panel is our regression estimate of the relationship between the level of consumer welfare and the total number of features used for personalization. We can think of the slope of this line as the posterior belief about the welfare effects of data granularity. Once again, we see a stark difference between the inequality-neutral ($r = 1$) and inequality-averse welfare functions ($r \in \{-1, 0\}$). The former, which only considers total surplus, implies a negative relationship between consumer welfare and the granularity of the targeting data. However, the inequality-averse welfare functions tend to favor personalized pricing and more data granularity. While this analysis does not provide a definitive case for or against the welfare effects of data granularity, it does confirm the need for more academic discourse on how to think about consumer welfare in the context of empirically-realistic models of heterogeneous demand when a representative consumer framework is untenable.

In spite of these nuances, we believe our results should challenge the prior that personalized prices are *per se* bad from a consumer point of a view. To be clear, we are not advocating for personalization as welfare-increasing. Rather, we believe the evidence suggests that data and privacy policies that treat personalized pricing as per se harmful may have unintended consequences and warrant further study.

7 Conclusions

A long theoretical literature has studied the welfare implications of monopoly price discrimination. In the digital era, large-scale price discrimination is becoming an empirical reality, raising an important public policy debate about the role of consumer information and its potential impact on consumer well-being. In our case study, we find that personalized pricing using machine learning increases firm profits by over 10% relative to uniform pricing, both in and out of sample, even when we cap the prices at $499. On the demand side, we find that personalized pricing reduces total consumer surplus. However, we also find that certain data policies that would restrict the use of specific consumer variables for targeting purposes could in fact exacerbate rather than offset the declines in consumer welfare. In our case study, we also find that the majority of consumer would benefit from being charged lower prices than the uniform rate even though total consumer surplus declines. Under standard alternative consumer welfare functions that value the allocation of surplus in addition to the level, we find that the allocative benefits of personalization (through a reduction in inequality) can outweigh the loss in total surplus. These allocative benefits accrue primarily to smaller firms.

The current public policy debate surrounding the fairness of differential pricing might consider the redistributive aspects of personalized pricing in addition to the total surplus implications. In addition, over-regulation of the types of data firms can use for personalized pricing purposes could exacerbate rather than offset some of the harm to consumers. For instance, we find instances of a non-monotonic relationship between consumer welfare and the total number of feature variables available for price-targeting purposes.

The results presented herein are based on a single case study of a large digital human resources platform with enterprise consumers. The generalizability of our findings may be limited beyond settings where, like ours, consumers are unlikely to be able to game the personalizing structure. We assume that consumers are unable to misrepresent their "types" to obtain lower prices (e.g., Acquisti and Varian, 2005; Fudenberg and Villas-Boas, 2006; Bonatti and Cisternas, 2018). Our findings also do not consider the potential role of longer-term consumer backlash based on subjective fairness concerns regarding differential pricing, which could lead to more price elastic demand in the long run under personalized pricing. This type of backlash might be more problematic in a consumer goods market where personalized pricing may be more transparent and less accepted[27]. Finally, our findings focus on the monopoly price discrimination problem for Ziprecruiter.com. We do not consider the impact of personalized pricing in a competitive market, where the potential toughening or softening of price competition would also impact the welfare implications[28].

In addition, our study was conducted in the context of a business-to-business digital platform selling to enterprise customers. An important direction for future research will be the study of personalized pricing in the context of consumer goods and the welfare implications for consumers with different incomes and socio-economic status.

[27]Negotiated price deals are quite common in B2B pricing, especially with sales agents.

[28]See for instance the empirical analysis of competitive geographic price discrimination in Dubé, Fang, Fong, and Luo (2017), the theoretical work by Corts (1998) and literature survey in Stole (2007)

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	Uniform	Perfect PD	Personalized 1 $\mathbf{D} = (\{1\}$ $\{2,3,4,5,6\})$	Personalized 2 $\mathbf{D} = (\{1\},$ $\{2,3\},$ $\{4,5,6\})$
prices	$p_i^U = \$4, \forall$i	$p_i^{PD} = i, \forall$i	$p_{\{1\}}^{PP1} = \$1$ $p_{\{2,3,4,5,6\}}^{PP1} = \$4$	$p_{\{1\}}^{PP2} = \$1$ $p_{\{2,3\}}^{PP2} = \$2$ $p_{\{4,5,6\}}^{PP2} = \$4$
profits	\$12	\$21	\$13	\$17
CS	\$3	0	\$3	\$4

Table 1: Data and Welfare

	Monthly Price
Control	99
Test 1	19
Test 2	39
Test 3	59
Test 4	79
Test 5	159
Test 6	199
Test 7	249
Test 8	299
Test 9	399

Table 2: Experimental Price Cells for Stage One

Feature Name
job state
company type
commissions offered
Number of job slots needed
total benefits
employment type
resume required
medical benefit
dental benefit
vision benefit
life insurance benefit
job category

Table 3: Company/Job Variables

Model	**In-Sample BIC**	Out-of-Sample RMSE	Out-of-Sample Hit Rate
MLE	10,018.78	0.412	70.3%
Lasso	8,366.47	0.410	76.9%
WLB range	(7,805.11 , 8,940.06)	0.405	76.9%

Table 4: Predictive Fit from MLE, Lasso and Weighted Likelihood Bootstrap estimation (WLB) (for WLB we report the range across all 100 bootstrap replications). In-Sample results are based on entire September 2015 sample with 7,866 firms. Out-of-Sample results are based on a randomly-selected (without replacement) training sample representing 90% of the firms, and a hold-out sample with the remaining 10% of the firms.

Price ($)	Acquisition	at least 1 month	at least 2 months	at least 3 months	at least 4 months
19	0.36	0.8	0.77	0.61	0.56
39	0.32	0.75	0.73	0.52	0.47
59	0.27	0.65	0.63	0.49	0.4
79	0.29	0.69	0.64	0.5	0.39
99	0.24	0.69	0.66	0.48	0.38
159	0.2	0.63	0.61	0.43	0.34
199	0.18	0.56	0.5	0.31	0.19
249	0.17	0.63	0.59	0.39	0.27
299	0.13	0.58	0.53	0.35	0.29
399	0.11	0.54	0.52	0.37	0.25

Table 5: Acquisition and Retention Rates (September 2015)

Decision-Theoretic Pricing Structures	**Price**	**Expected Conversion Rate**		**Expected Revenue ($ per Consumer)**	
		Mean	**95% Cred. Int.**	**Mean**	**95% Cred. Int.**
Control	$99	0.25	(0.23,0.28)	25.09	(23.02,27.48)
Uniform	$327	0.12	(0.1,0.14)	39.01	(31.9,46.58)
Personalized	($126,6292)	0.12	(0.1,0.14)	46.57	(32.89,60.8)
Personalized ($499 price cap)	($126,$499)	0.13	(0.11,0.16)	42.21	(33.41,50.53)

Table 6: Posterior expected conversion and revenue per consumer by pricing structure for September 2015 experiment.

	control ($99)	test ($249)	test (personalized pricing)
Sample Size	1,360	1,430	2,485
mean conversion	0.23	0.15	0.15
	(0.21,0.25)	(0.13,0.17)	(0.13,0.16)
mean revenue per consumer	$22.57	$37.79	$41.59
	(20.36,24.77)	(33.15,42.42)	(37.49,45.7)
posterior mean conversion	0.26	0.15	0.14
	(0.23,0.29)	(0.13,0.18)	(0.12,0.17)
posterior mean revenue per consumer	$25.5	$38.37	$41.05
	(23.26,28.31)	(32.04,44.9)	(33.78,48.78)

Table 7: Predicted versus Realized Outcomes in November 2015 Experiment (Below each realized outcome, we report in brackets the 95% confidence intervals. Below each posterior predicted outcome, we report in brackets the 95% credibility interval.)

Table 8: Consumer Welfare and Data based Pricing

(a) Comparing Theoretically Optimal Pricing Policies

Measure	r	$S_r(\mathbf{p}^{pers})$	$S_r(\mathbf{p}^{unif})$	$\Delta = S_r(\mathbf{p}^{pers}) - S_r(\mathbf{p}^{unif})$	$\%\Delta S_r(\mathbf{p})$
Harmonic Mean	-1	46.8255	33.6011	13.2244	39.36
Geometric Mean	0	58.2786	57.5773	0.70127	1.22
Arithmetic Mean	$+1$	71.4094	95.2247	-23.8153	-25.01

(b) Implemented Personalized vs. Optimal Uniform

Measure	r	$S_r(\mathbf{p}^{pers})$	$S_r(\mathbf{p}^{unif})$	$\Delta = S_r(\mathbf{p}^{pers}) - S_r(\mathbf{p}^{unif})$	$\%\Delta S_r(\mathbf{p})$
Harmonic Mean	-1	50.1969	33.6011	16.5958	49.39
Geometric Mean	0	67.3144	57.5773	0.7371	16.91
Arithmetic Mean	$+1$	93.2841	95.2247	-1.9406	-2.04

(c) Implemented Personalized vs. Implemented Uniform Pricing Policies

Measure	r	$S_r(\mathbf{p}^{pers})$	$S_r(\mathbf{p}^{unif})$	$\Delta = S_r(\mathbf{p}^{pers}) - S_r(\mathbf{p}^{unif})$	$\%\Delta S_r(\mathbf{p})$
Harmonic Mean	-1	50.1969	43.4767	6.7202	15.46
Geometric Mean	0	67.3144	68.1889	-0.8745	-1.28
Arithmetic Mean	$+1$	93.2841	105.3496	-12.0656	-11.45

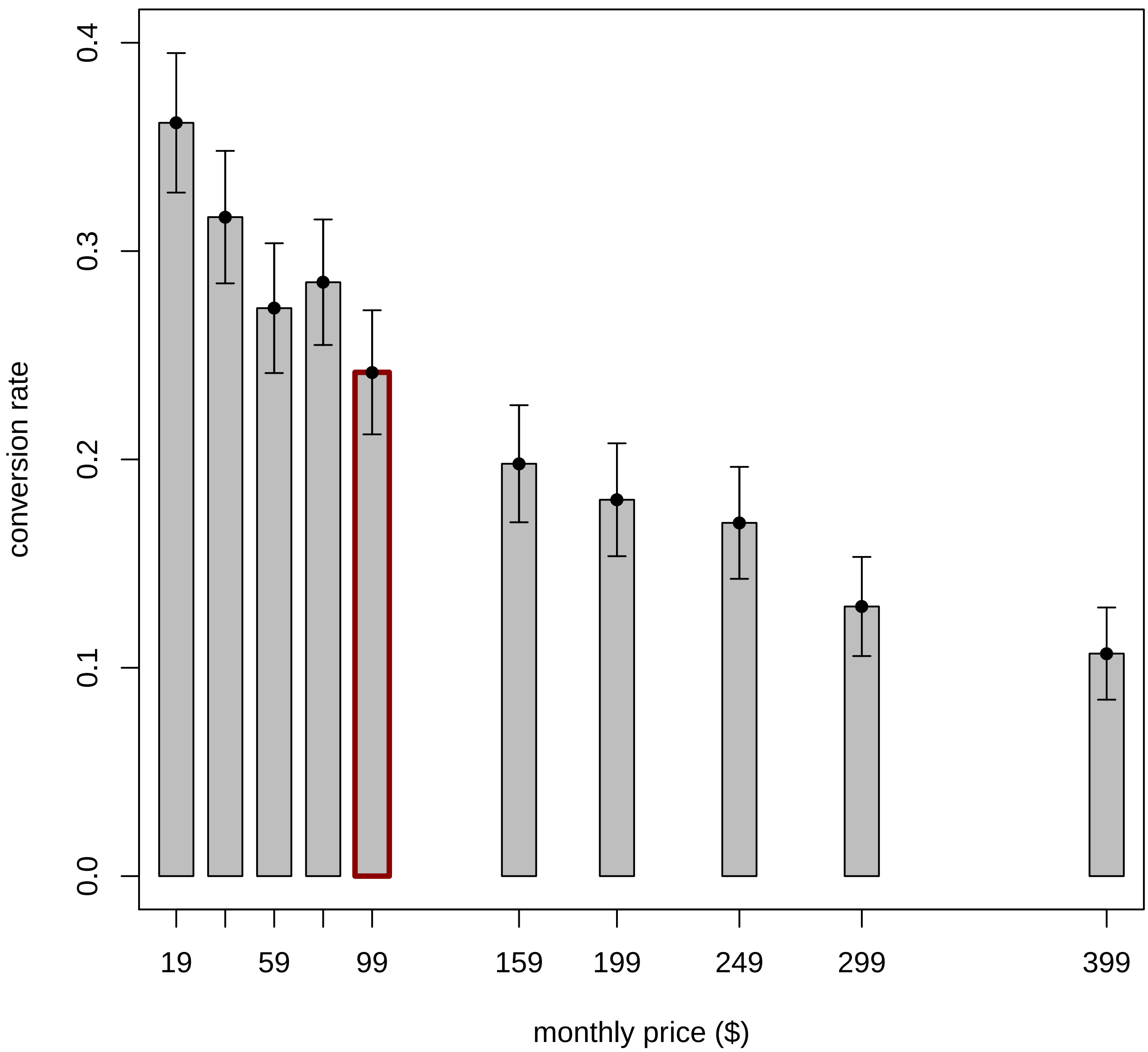


Figure 1: Stage One Experimental Conversion Rates. Each bar corresponds to one of our 10 experimental price cells. The height of the bar corresponds to the average conversion rate within the cell. Error bars indicate the 95% confidence interval for the conversion rate.

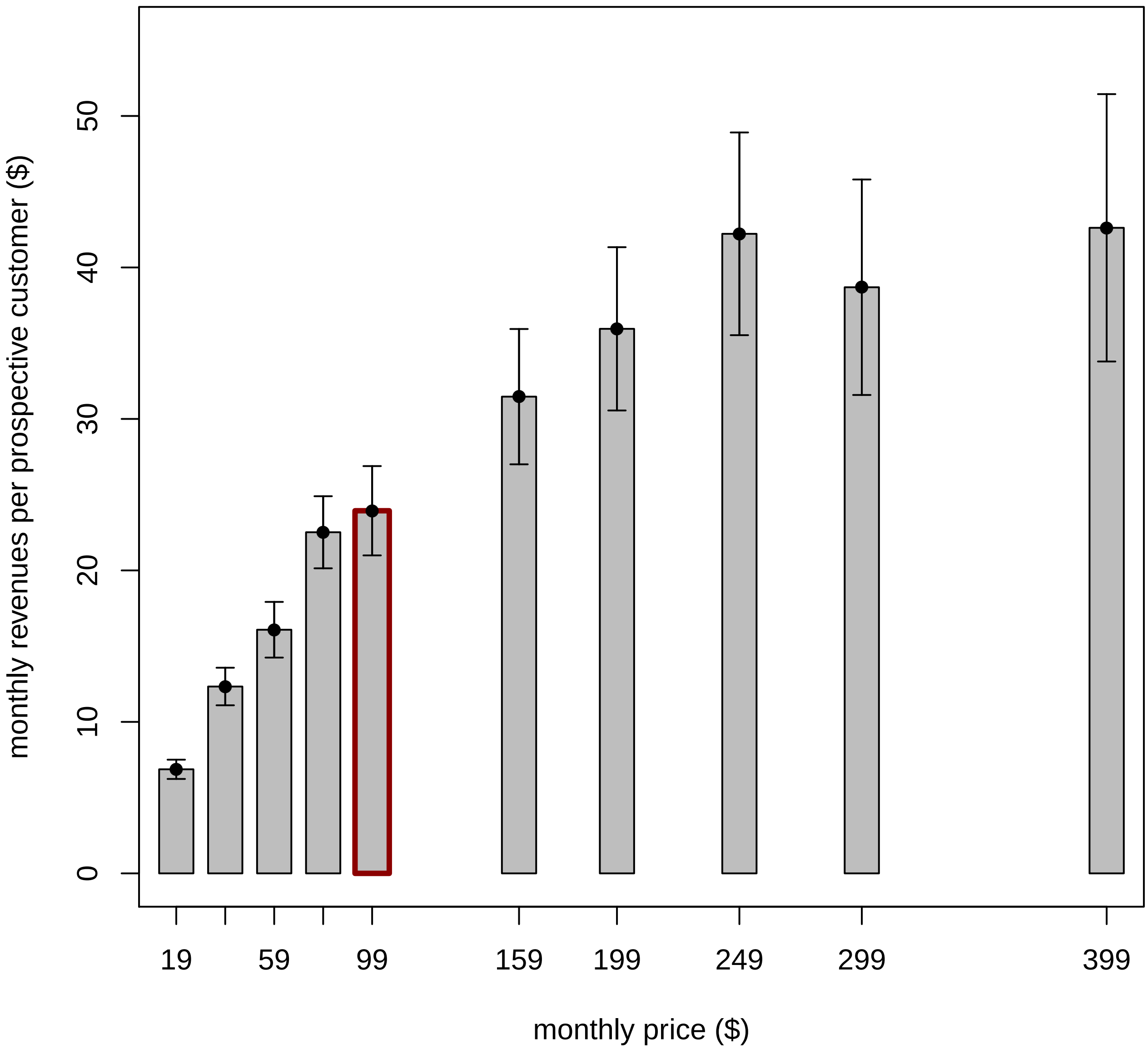


Figure 2: Stage One Experimental Revenues per Customer. Each bar corresponds to one of our 10 experimental price cells. The height of the bar corresponds to the average revenue per prospective consumer within the cell. Error bars indicate the 95% confidence interval for the revenues per consumer.

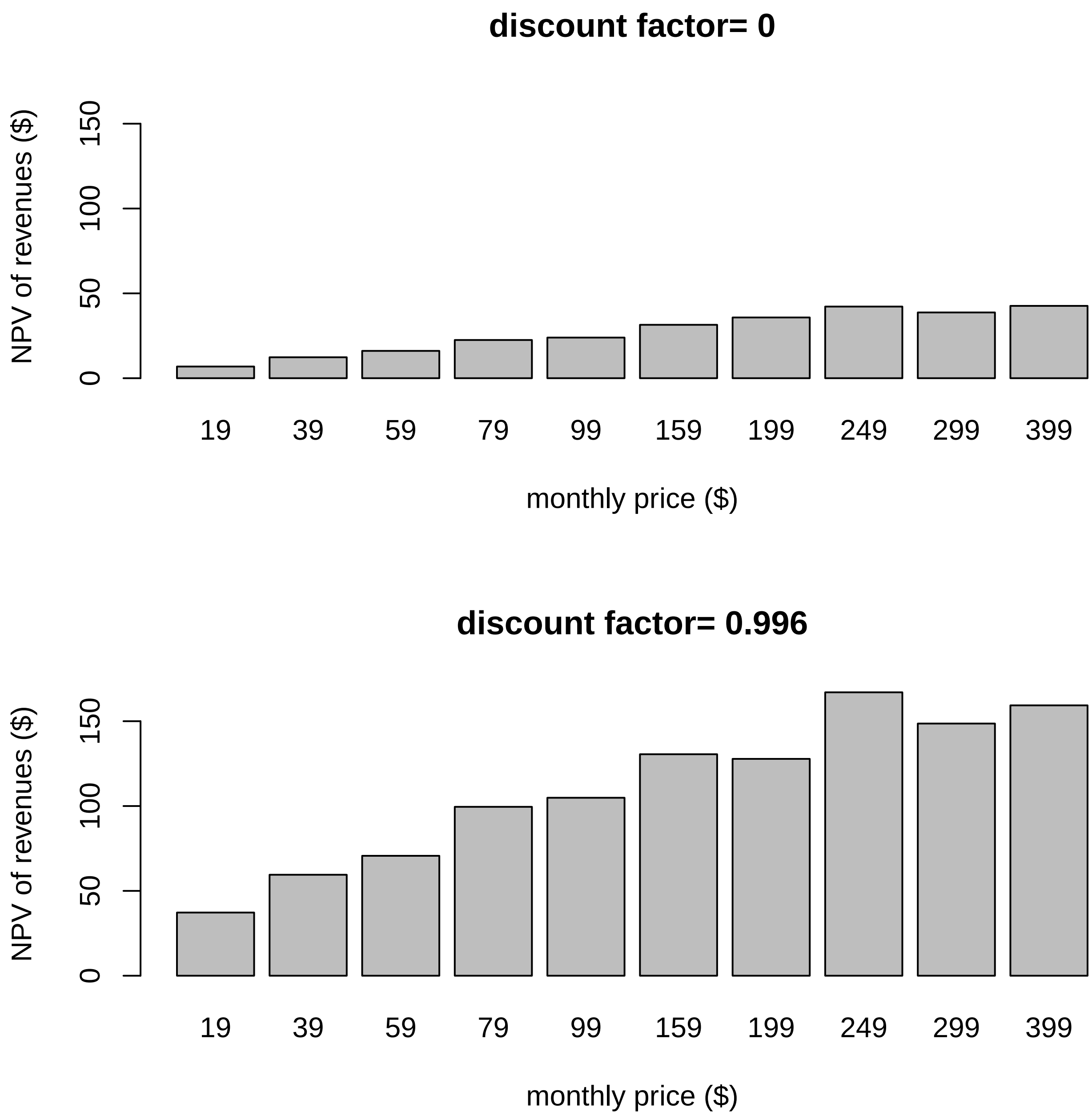


Figure 3: Expected Net Present Value of Monthly Revenues Per Lead over a 4-Month Horizon (September 2015)

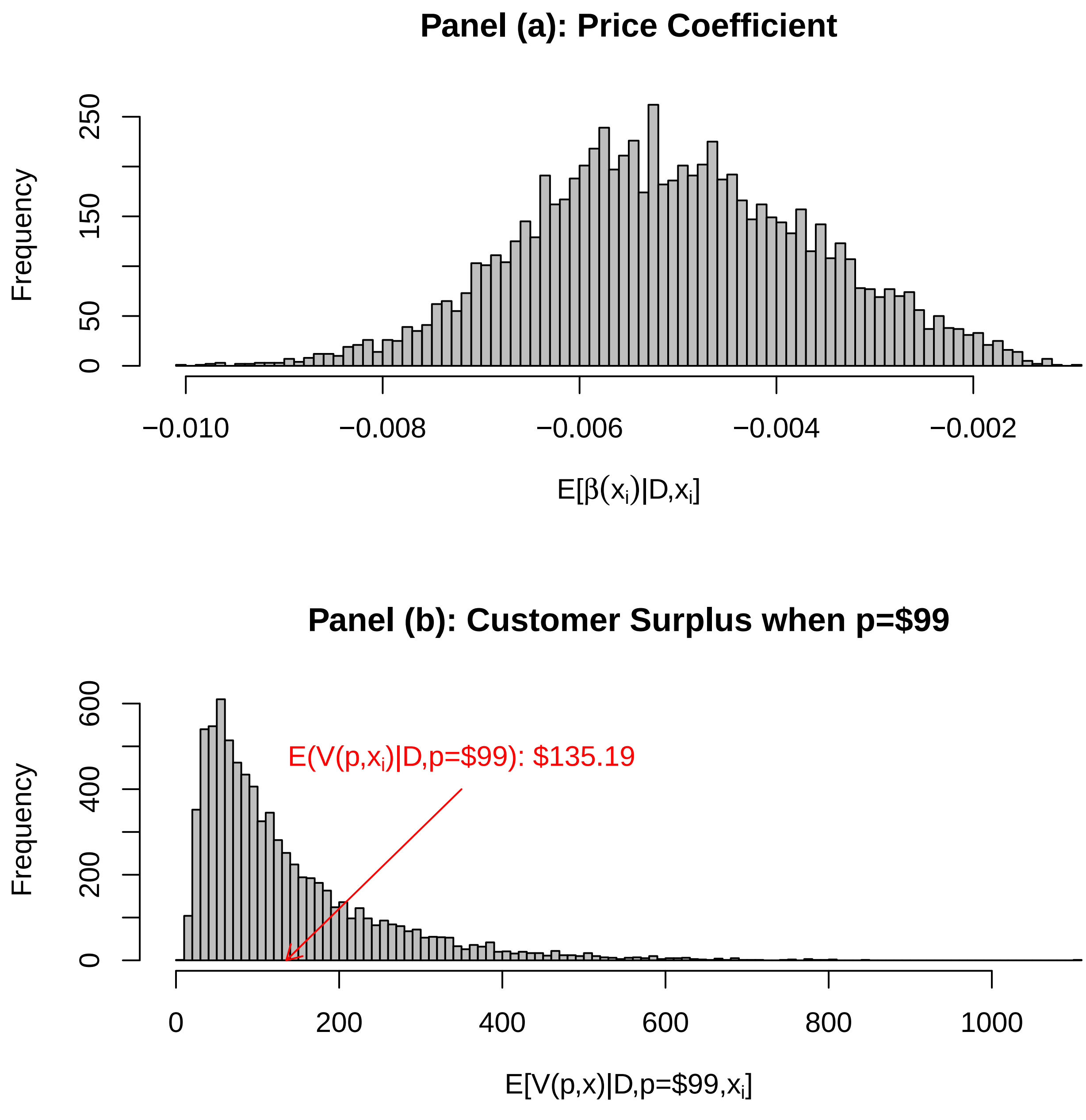


Figure 4: Distribution across consumers of posterior mean price sensitivity and posterior surplus from the provision of the service (N=7,867).

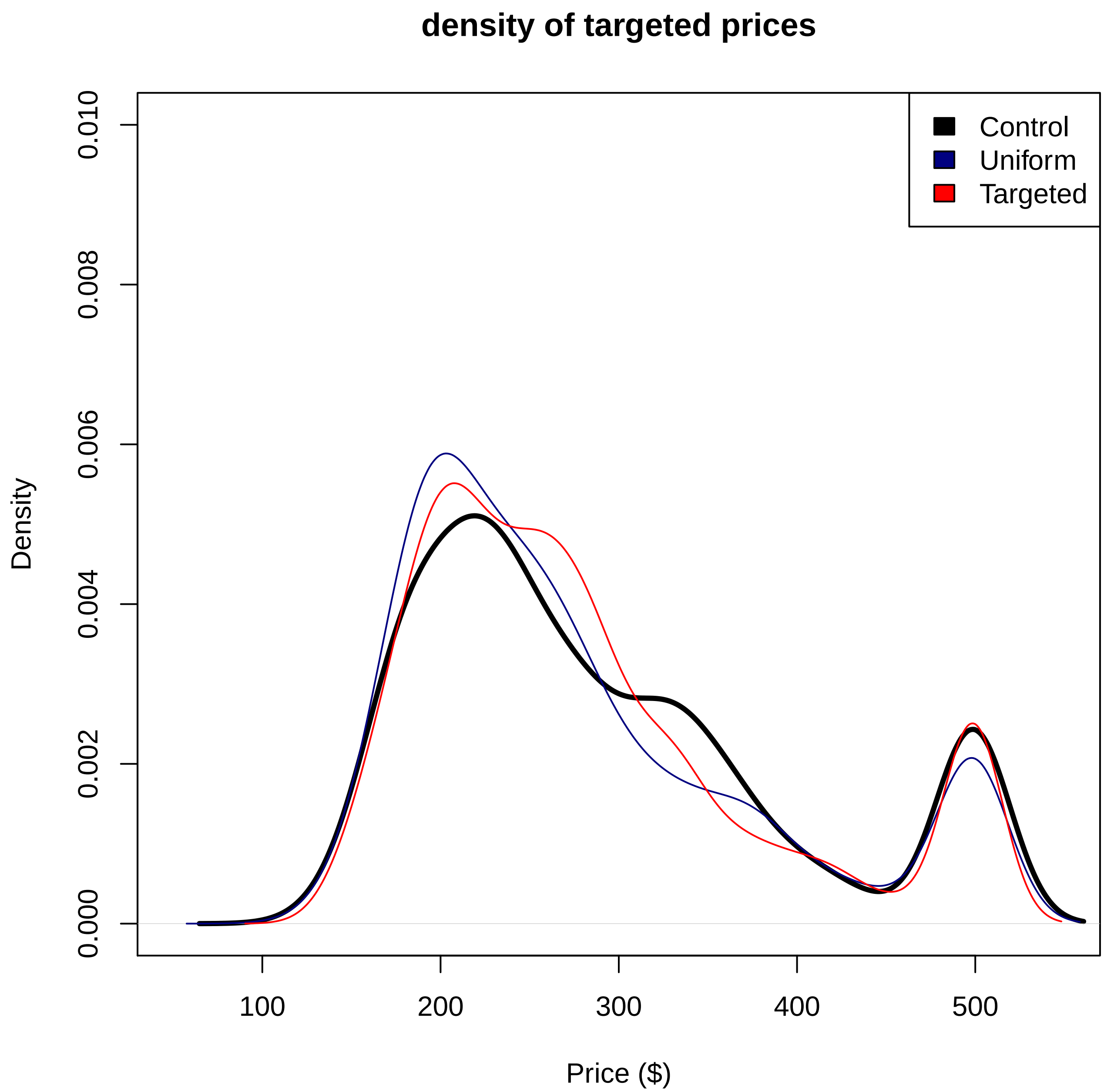
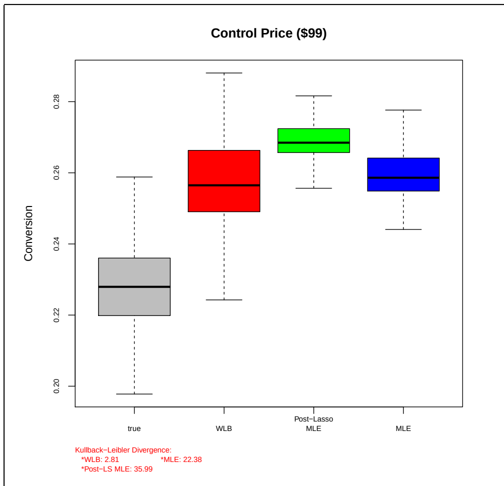
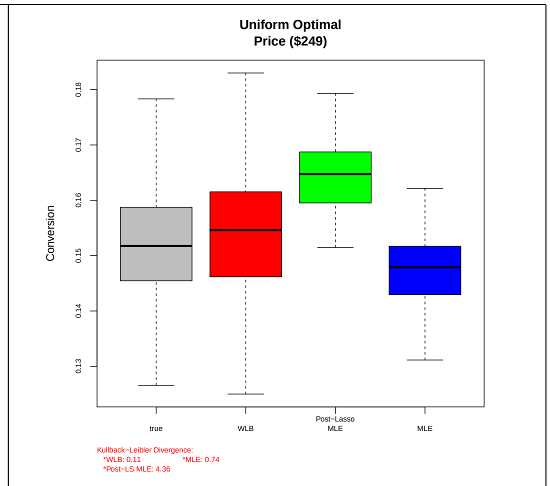


Figure 5: Density of Targeted Prices in Each Cell (November, 2015). For each of the cells, we plot the estimated density using a Gaussian kernel.

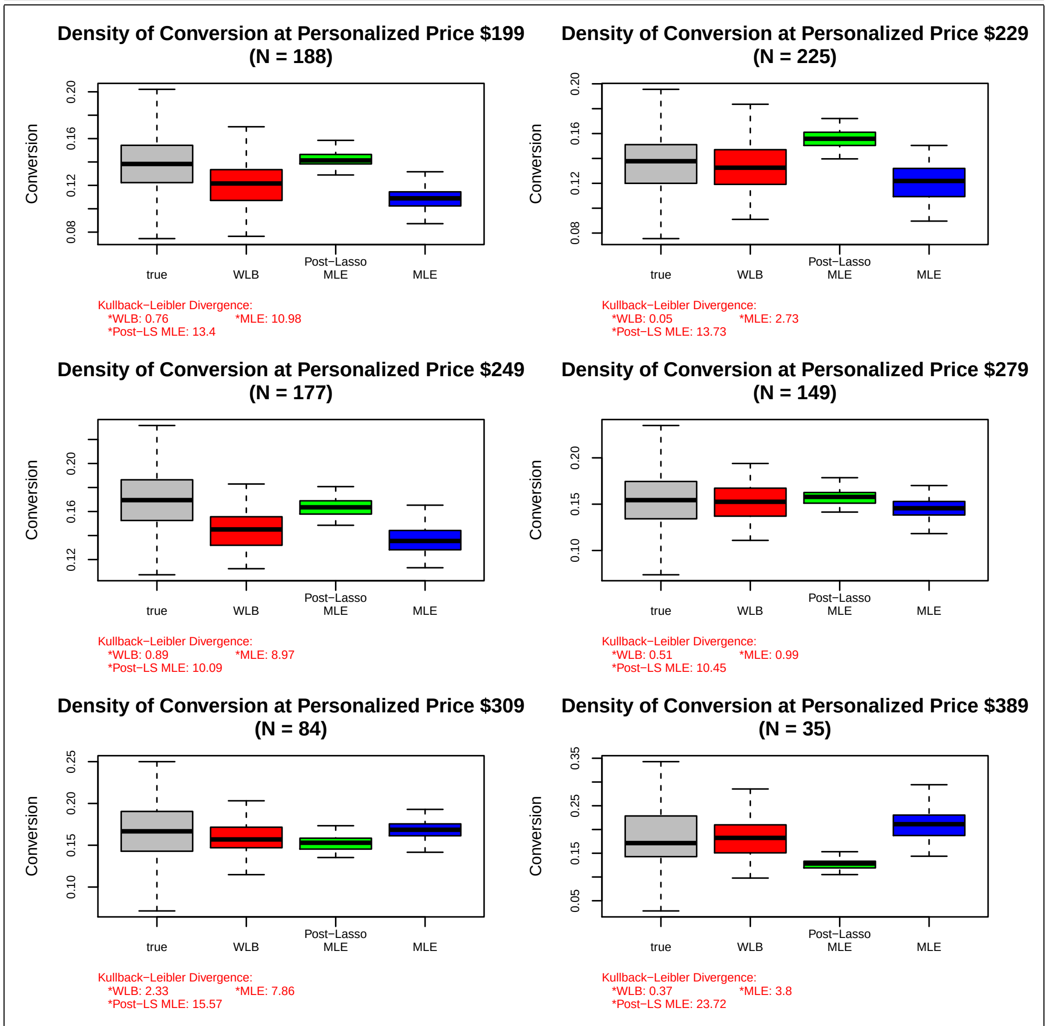
Figure 6: Comparison of Predicted and Realized Conversion



(a) Control Pricing ($99)



(b) Uniform Optimal Pricing



(c) scalable price targeting

The plots compare the empirical density of realized conversion, for a given pricing structure, to the corresponding predicted densities for WLB, post-Lasso MLE and MLE respectively. The density of realized conversions is computed by bootstrapping (with replacement) from the Nov data.

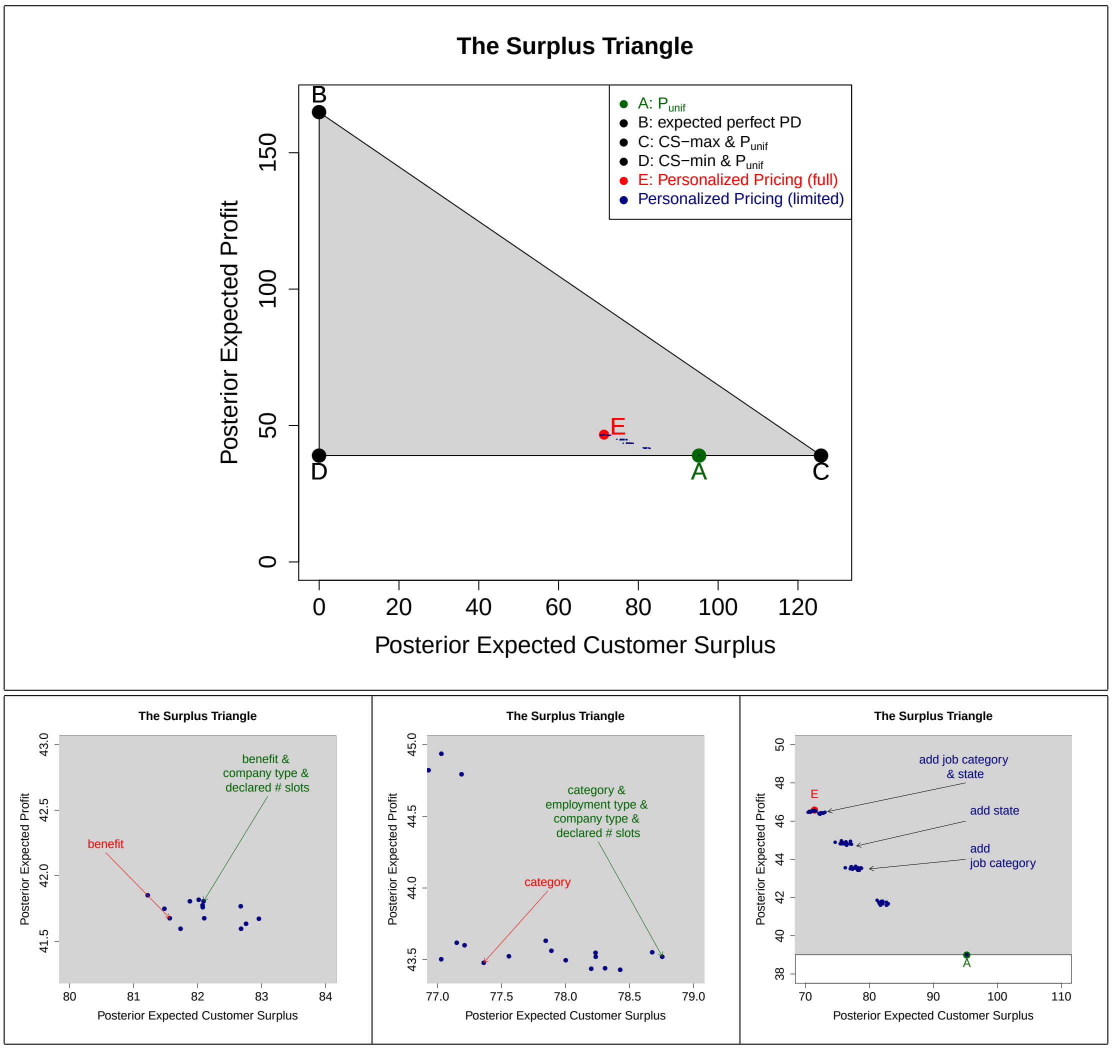


Figure 7: Surplus Triangle

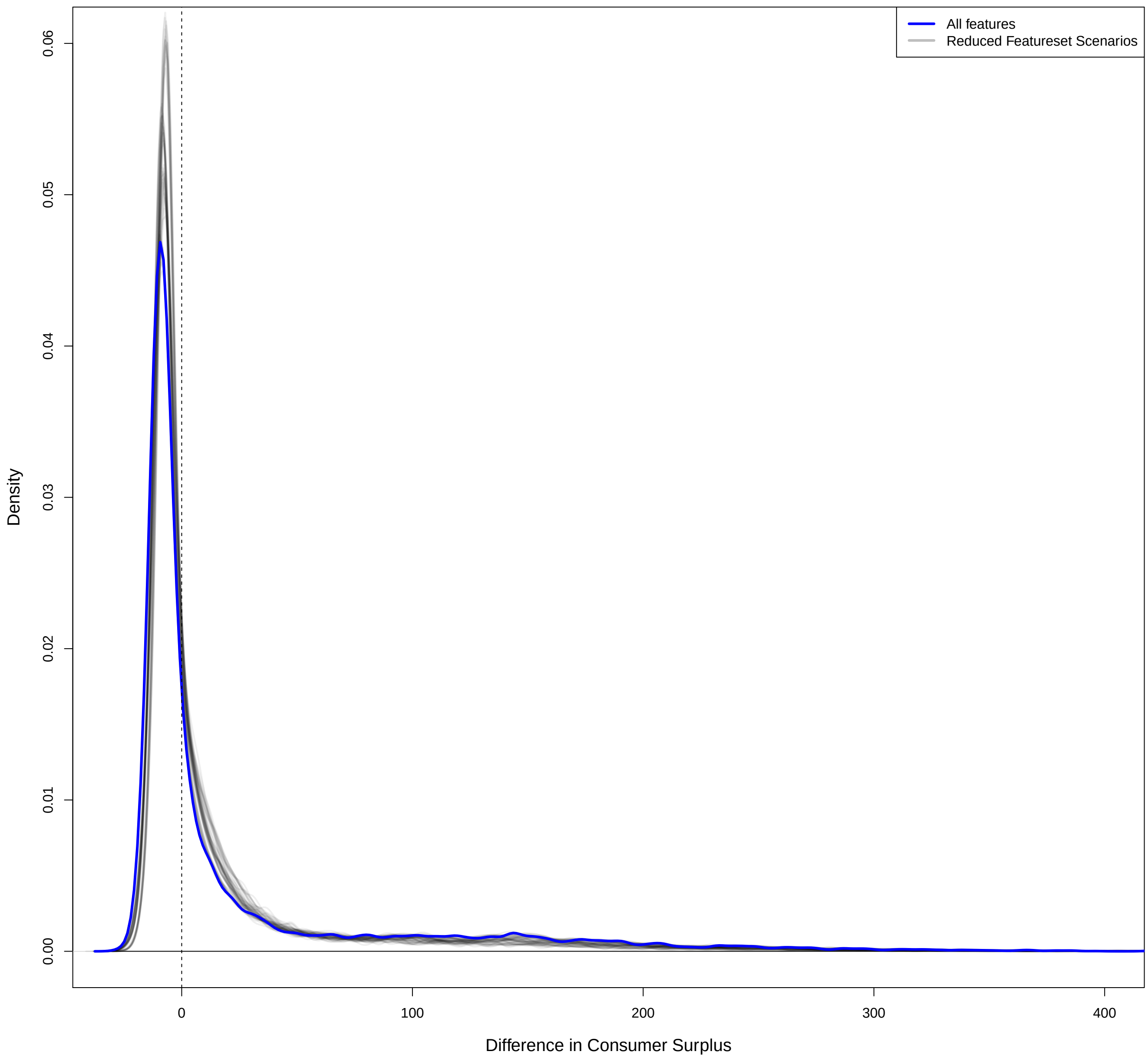


Figure 8: Densities of the Change in Expected Posterior Surplus Across Customers Under Personalized Pricing versus Uniform Pricing. We report densities for all reduced featureset database scenarios, each in gray. We highlight the main case that uses all the features in blue.

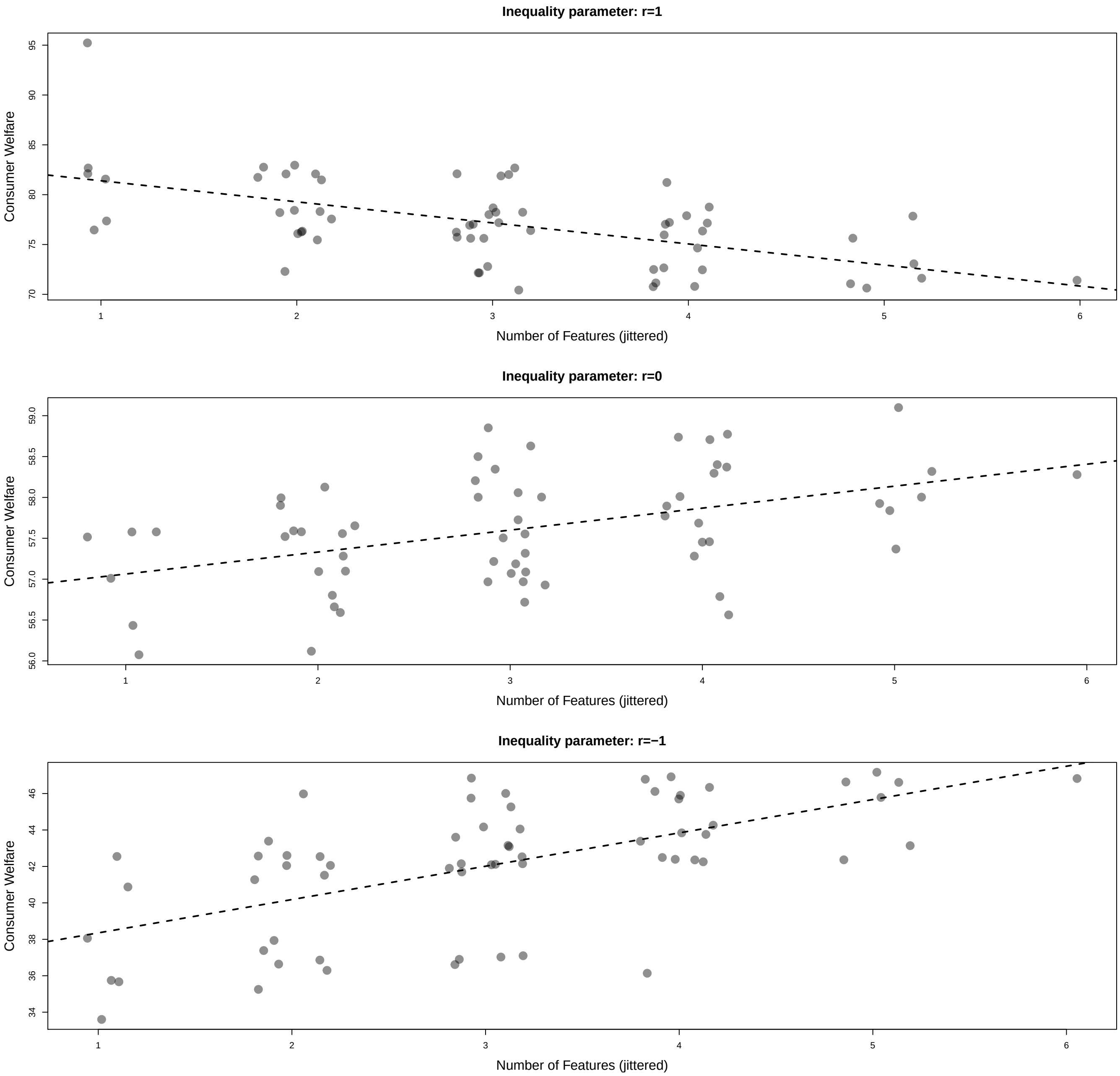
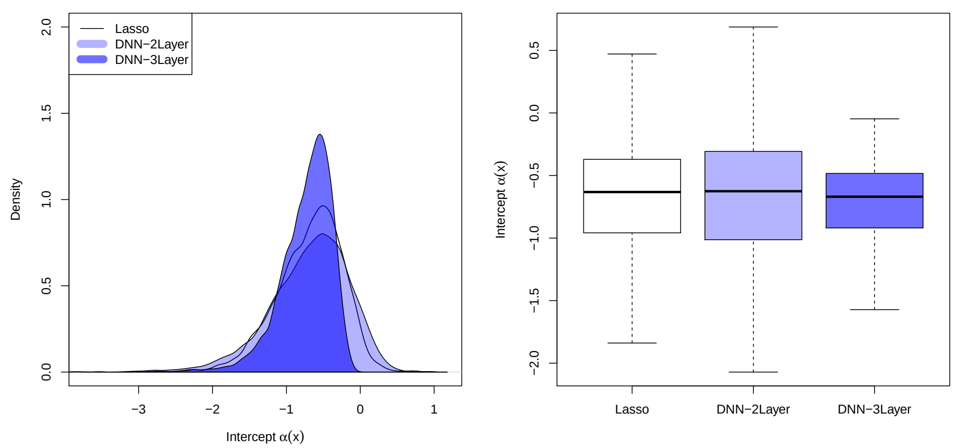


Figure 9: Difference in Consumer surplus vs. Number of Features used

Figure 10: Comparison of Individual Posterior Means of Parameters

(a) Intercepts ($\alpha(x)$)



(b) Price Coefficients ($\beta(x)$)

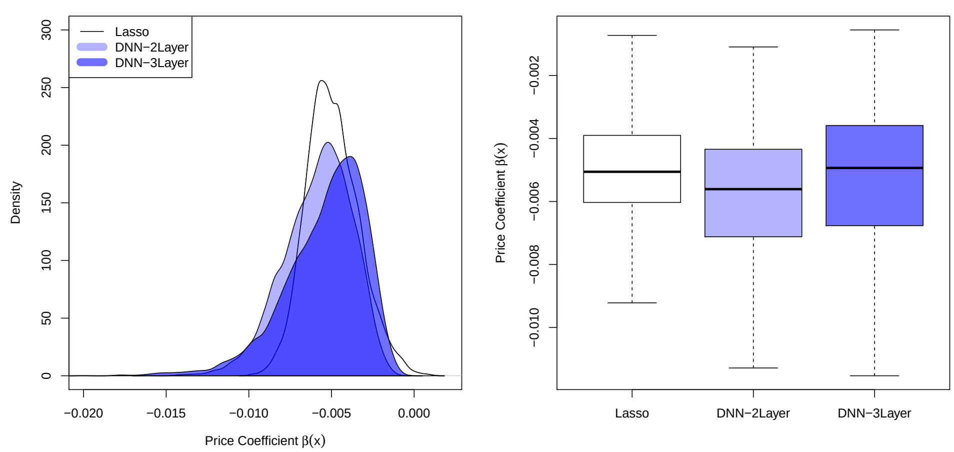


Figure 11: Comparison of Personalized Prices $(p^* (x))$

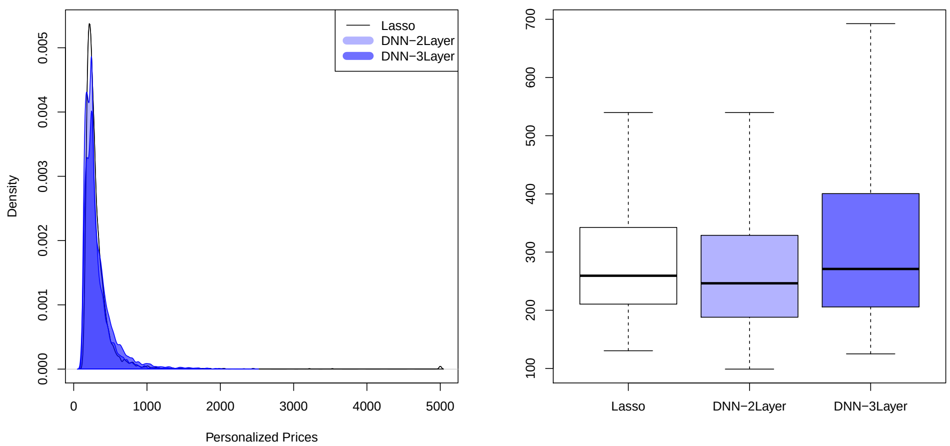
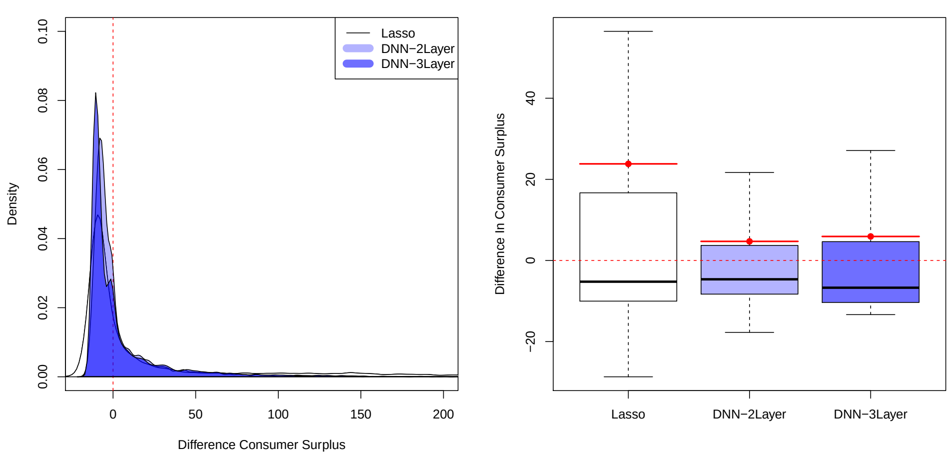


Figure 12: Comparison of Differences in Consumer Surplus



A The Bayesian Lasso

We start with our regularization procedure. Following Tibshirani (1996), suppose each model parameter, Θ_j, is assigned an i.i.d. Laplace prior with scale $\tau > 0$: $\Theta_j \sim La(\tau)$ where $\tau = N\lambda$. We can write the the posterior distribution of Θ analytically:

$$F_{\Theta}(\Theta|\boldsymbol{D}) \propto \ell(\boldsymbol{D}|\Theta) - \sum_{j=1}^{J} \tau_j |\Theta_j| \tag{18}$$

where $\ell(\boldsymbol{D}|\Theta)$ is the log-likelihood of the demand data as before. This framework is termed the Bayesian Lasso (Park and Casella 2008) on account of the Bayesian interpretation of the Lasso penalized objective function. The MAP (maximum a posteriori) estimator that optimizes (18) can be shown to be equivalent to the Lasso regression:

$$\Theta^{Lasso} = \underset{\Theta \in \mathbb{R}^J}{argmax} \left\{ \ell(\boldsymbol{D}|\Theta) - N\lambda \sum_{j=1}^{J} |\Theta_j| \right\}. \tag{19}$$

In Appendix C, we describe the path-of-one-step estimators procedure used to select λ and generate estimates of Θ and its sparsity structure (see also Taddy (2015b)).

B The Weighted Likelihood Bootstrap

While the MAP estimator generates a point estimate of the posterior mode it does not offer a simple way to calibrate the uncertainty in these estimates. Park and Casella (2008) propose a Gibbs sampler for a fully Bayesian implementation of the Lasso, but the approach would not scale well to settings with very large-dimensional x_i[29]. Instead, we simulate the approximate posterior using a Weighted Likelihood Bootstrap (WLB) of the Lasso problem. The Weighted Likelihood Bootstrap (Newton and Raftery (1994)) is an extension of the Bayesian Bootstrap originally proposed by Rubin (1981).[30] As discussed in Efron (2012), the BB and the WLB are computationally simple alternatives to MCMC approaches. In our context, the approach is scalable to settings with a large-dimensional parameter space, and is relatively fast, making consumer classification and price discrimination practical to implement in real time. Conceptually, the approach consists of drawing weights associated with the observed data sample and solving a weighted version of (19).

[29] Challenges include drawing from a large-dimensional distribution, assessing convergence of the MCMC sampler, tuning the algorithm and storing a non-sparse simulated chain in memory.

[30] To be clear, our implementation only uses the first stage of the WLB procedure described in Newton and Raftery (1994) and does not implement the Sampling-Importance-Resampling (SIR) stage. Newton and Raftery (1994) show that the first stage is sufficient to obtain a first order approximation of the posterior. We could also describe our implementation simply as a variant of the Bayesian Bootstrap but we chose to call it the WLB to acknowledge the contribution of Newton and Raftery (1994) who first outlined the possibility of recasting the Rubin (1981) framework of using the weighted likelihoods.

The application of Lasso to each replication ensures a sparsity structure that facilitates the storage of the draws in memory. This is a promising approach to approximating uncertainty in complex econometric models (see e.g. Chamberlain and Imbens (2003)).

We construct a novel WLB type procedure to derive the posterior distribution of $\hat{\Theta}|_{\lambda^*}$, $F(\Theta)$. Consider our data sample $\mathbf{D} = (D_1, ..., D_N)$. We assume that the data-generating process for $\boldsymbol{D}$ is discrete with support points $(\zeta_1, ..., \zeta_L)$ and corresponding probabilities $\phi = (\phi_1, ..., \phi_L)$: $Pr(D_i = \zeta_l) = \phi_l$. We can allow L to be arbitrarily large to allow for flexibility in this representation. We assume the following Dirichlet prior on the probabilities

$$\phi \sim Dir(\mathbf{a}) \propto \prod_{l=1}^{L} \phi_l^{a_l - 1}, \ a_l > 0.$$

Following the convention in the literature, we use the improper prior distribution with $a_l \to 0$. This assumption implies that any support points, ζ_l, not observed in the data will have $\phi_l = 0$ with posterior probability one: $Pr(\phi_l = 0) = 1, \ \forall \zeta_l \notin \mathbf{D}$. This prior is equivalent to using the following independent exponential prior: $V_l \sim Exp(1)$ where $V_l = \sum_{k=1}^{L} \phi_k \phi_l$.

We can now write the posterior distribution of observing a given data point, D as follows

$$f(D) = \sum_{i=1}^{N} V_i \mathbb{1}_{\{D=\zeta_i\}}, \ V_i \sim i.i.d.Exp(1).$$

The algorithm is implemented as follows. For each of the bootstrap replications $b = 1, ..., B$:

1. Draw weights: $\left\{V_i^b\right\}_{i=1}^N \sim Exp(\mathbf{1}_N)$
2. Run the Lasso

   $$\hat{\Theta}^b|_\lambda = \underset{\Theta \in \mathbb{R}^J}{argmin} \left\{ \ell^b(\Theta) + N\lambda \sum_{j=1}^{J} |\Theta_j| \right\}$$

   where $\ell^b(\boldsymbol{D}|\Theta) = \sum_{i=1}^{N} V_i^b \ell(D_i|\Theta)$, using the algorithm (21) in Appendix C

   (a) Construct the regularization path, $\left\{\hat{\Theta}^b|_\lambda\right\}_{\lambda=\lambda_1}^{\lambda_T}$

   (b) Use k-fold-cross validation to determine the optimal penalty, λ^*
3. Retain $\hat{\Theta}^b \equiv \hat{\Theta}^b|_{\lambda^{b*}}$.

We can then use the bootstrap draws, $\left\{\hat{\Theta}^b\right\}_{b=1}^B$, to simulate the posterior of interest, $F_\Psi(\Psi_i)$. We construct draws $\left\{\Psi_i^b\right\}_{b=1}^B$, where $\Psi_i^b = \Psi\left(x_i; \Theta^b\right)$, which can be used to simulate the posterior $F_\Psi(\Psi_i)$. We will use this sample to quantify the uncertainty associated with various functions of Ψ_i such as profits and demand elasticities.

C Appendix: Lasso Regression

The penalized Lasso estimator solves for

$$\hat{\Theta}|_{\lambda} = \underset{\Theta \in \mathbb{R}^J}{argmin} \left\{ \ell\left(\Theta\right) + N\lambda \sum_{j=1}^{J} |\Theta_j| \right\} \tag{20}$$

where $\lambda > 0$ controls the overall penalty and $|\Theta_j|$ is the L_1 coefficient cost function. Note that as $\lambda \to 0$, we approach the standard maximum likelihood estimator. For $\lambda > 0$, we derive simpler "regularized" models with low (or zero) weight assigned to many of the coefficients. Since the ideal λ is unknown a priori, we derive a regularization path, $\left\{\hat{\Theta}|_{\lambda}\right\}_{\lambda=\lambda_1}^{\lambda_T}$, consisting of a sequence of estimates of Θ corresponding to successively lower degrees of penalization. Following Taddy (2015b), we use the following algorithm to construct the path:

1. $\lambda_1 = inf\left\{\lambda : \hat{\Theta}|_{\lambda_1} = 0\right\}$

2. set step size of $\delta \in (0, 1)$

3. for $t = 2, ..., T$:

$$\begin{aligned} \lambda^t &= \delta\lambda^{t-1} \\ \omega_j^t &= \left(|\Theta_j^{t-1}|\right)^{-1}, \; j \in \hat{S}_t \\ \hat{\Theta}^t &= \underset{\Theta \in \mathbb{R}^J}{argmin} \left\{ \ell\left(\Theta\right) + N \textstyle\sum_{j=1}^{J} \lambda^t \omega_j^t |\Theta_j| \right\}. \end{aligned} \tag{21}$$

The algorithm produces a weighted-L_1 regularization, with weights ω_j. The concavity ensures that the weight on the penalty on $\hat{\Theta}_j^t$ falls with the magnitude of $|\hat{\Theta}_j^t|$. As a result, coefficients with large values earlier in the path will be less biased towards zero later in the path. This bias diminishes faster with larger values of γ.

The algorithm in 21 above generates a path of estimates corresponding to different levels of penalization, λ. We use K-fold cross-validation to select the "optimal" penalty, λ^*. We implement the approach using the *cv.gamlr* function from the *gamlr* package in **R**.

D The Deep Learning Framework

The deep learning estimator follows the WLB procedure for the Lasso described in section (B). The key point of departure is in the definition of the loss function. We use $\ell^b_{\text{DNN}}(\Theta)$ to denote the logit loss function and we approximate the structural parameters $\{\alpha(x), \beta(x)\}$ with deep neural networks (DNNs). The architecture of the DNNs are chosen to match the complexity levels to the data (see Farrell, Liang, and Misra (2021b) and Farrell, Liang, and Misra (2021a) for a more in-depth discussion). For the analysis presented herein, we used two specifications of the network with two and three hidden layers of ten nodes each. The total number of parameters in each of these models is 1440 and 2340, respectively. The minimization procedure was coded in Tensorflow (Abadi, Agarwal, Barham, Brevdo, Chen, Citro, Corrado, Davis, Dean, Devin, Ghemawat, Goodfellow, Harp, Irving, Isard, Jia, Jozefowicz, Kaiser, Kudlur, Levenberg, Mané, Monga, Moore, Murray, Olah, Schuster, Shlens, Steiner, Sutskever, Talwar, Tucke (2015)) and used the default specification of the ADAM learning algorithm (Kingma and Ba (2015)).

The algorithm is implemented as follows. For each of the bootstrap replications $b = 1, ..., B$:

1. Draw weights: $\left\{V^b_i\right\}^N_{i=1} \sim Exp\left(\mathbf{1}_N\right)$
2. Run the DNN learning algorithm to obtain

$$\hat{\Theta}^b_{\text{DNN}} = \underset{\Theta \in \mathbb{R}^J}{argmin} \left\{\ell^b_{\text{DNN}}(\Theta)\right\}$$

   where $\ell^b_{\text{DNN}}(\boldsymbol{D}|\Theta) = \sum^N_{i=1} V^b_i \ell_{\text{DNN}}(D_i|\Theta)$.

3. Retain $\hat{\Theta}^b_{\text{DNN}}$.

As before, we use the bootstrap draws, $\left\{\hat{\Theta}^b_{\text{DNN}}\right\}^B_{b=1}$, to simulate the posterior of interest, $F_\Psi(\Psi_i)$. We construct draws $\left\{\Psi^b_i\right\}^B_{b=1}$, where $\Psi^b_i = \Psi\left(x_i; \Theta^b\right)$, which can be used to simulate the posterior $F_\Psi(\Psi_i)$.

E Appendix: Perfect Price Discrimination

Suppose the firm observed not only the full feature set for a consumer i, X_i,but also the random utility shock, ϵ_i. Under perfect price discrimination, the firm would set the personalized price

$$p^{PD}_i = \max\left(WTP_i, 0\right)$$

where WTP_i is consumer i's maximum willingness-to-pay (WTP)

$$WTP_i = \frac{\left(\alpha(X_i) + \epsilon_i\right)}{\beta(X_i)}. \tag{22}$$

Customer i would deterministically buy as long as $WTP_i \geq 0$.

Accounting for the fact that the researcher (unlike the firm in this case) does not observe ϵ, the expected probability that a consumer with preferences (α, β) would purchase at the perfect price discrimination price is

$$\begin{aligned} \mathbb{P}\left(p^{PD}; X_i, \Theta\right) &= Pr\left(WTP \geq 0\right) \\ &= 1 - \tfrac{1}{1+\exp(\alpha)}. \end{aligned} \tag{23}$$

The corresponding expected profit from this consumer is

$$\pi\left(p^{PD}|\alpha, \beta\right) = E\left(WTP|WTP \geq 0, \alpha, \beta\right) Pr\left(buy|p = p^{PD}\alpha, \beta\right) . \tag{24}$$

where

$$E\left(WTP|WTP > 0, \alpha, \beta\right) = \tfrac{\alpha}{\beta} + \tfrac{1}{\beta}\left(-\alpha + \tfrac{[1+\exp(\alpha)]ln[1+\exp(\alpha)]}{\exp(\alpha)}\right). \tag{25}$$

We now derive the result in 25. Recall the random utility shock is assumed to be i.i.d. logistic with PDF

$$f\left(\Delta\epsilon\right) = \frac{exp\left(-\Delta\epsilon\right)}{\left[1 + exp\left(-\Delta\epsilon\right)\right]^2}$$

and CDF

$$F\left(\Delta\epsilon\right) = \frac{1}{1 + exp\left(-\Delta\epsilon\right)}.$$

The truncated PDF for $\Delta\epsilon$ when it is known to be strictly greater than $k > 0$ is

$$f\left(\Delta\epsilon|\Delta\epsilon \geq k\right) = \frac{f\left(\Delta\epsilon\right)}{Pr\left(\Delta\epsilon \geq k\right)} = \left[\frac{exp\left(-k\right)}{1 + exp\left(-k\right)}\right]^{-1} \frac{exp\left(-\Delta\epsilon\right)}{\left[1 + exp\left(-\Delta\epsilon\right)\right]^2}$$

We can then compute the conditional expectation of the truncated random variable$\Delta\epsilon$ when $k > 0$ as follows:

$$\begin{aligned} E\left(\Delta\epsilon|\Delta\epsilon \geq k\right) &= \left[Pr\left(\Delta\epsilon \geq k\right)\right]^{-1} \int_k^{-\infty} \Delta\epsilon f\left(\Delta\epsilon\right) d\Delta\epsilon \\ &= \left[\tfrac{exp(-k)}{1+exp(-k)}\right]^{-1} \int_k^{-\infty} \Delta\epsilon \tfrac{exp(-\Delta\epsilon)}{[1+exp(-\Delta\epsilon)]^2} d\Delta\epsilon \\ &= \left[\tfrac{1+exp(-k)}{exp(-k)}\right]\left[\tfrac{kexp(-k)+[1+exp(-k)]ln[1+exp(-k)]}{1+exp(-k)}\right] \\ &= k + \tfrac{[1+exp(-k)]ln[1+exp(-k)]}{exp(-k)} \end{aligned}$$

where

$$\Delta\epsilon \frac{exp\left(-\Delta\epsilon\right)}{\left[1 + exp\left(-\Delta\epsilon\right)\right]^2} = \frac{d\left(-\frac{\Delta\epsilon e(-\Delta\epsilon)+[1+e(-\Delta\epsilon)]ln[1+e(-\Delta\epsilon)]}{[1+e(-\Delta\epsilon)]}\right)}{d\Delta\epsilon}.$$